EST. 2021 EMC
EDITORIAL MAR CARIBE

# GENERAL METHODOLOGICAL PROPOSALS FOR COMPUTER SCIENCE RESEARCH:

## Prototyping, mathematical modeling, and the scientific method

MARIA MAURA CAMAC TIZA - ERIKA JUANA ZEVALLOS VERA - OMAR TUPAC AMARU CASTILLO PAREDES - JUAN FRANCISCO RAMÍREZ VELIZ - ISMAEL EDWIN SALAZAR VILLAVICENCIO - JOSÉ CÉSAR PIEDRA ISUSQUI - LUIS ALBERTO JUSCAMAYTA RAMÍREZ

ISBN 978-9915-698-39-7

**General methodological proposals for Computer Science research: Prototyping, mathematical modeling, and the scientific method**

*Camac Tiza, Maria Maura; Zevallos Vera, Erika Juana; Castillo Paredes, Omar Tupac Amaru; Ramírez Veliz, Juan Francisco; Salazar Villavicencio, Ismael Edwin; Piedra Isusqui, José César; Juscamayta Ramírez, Luis Alberto*



First edition (1st ed.): October, 2025

Edited by:

**Editorial Mar Caribe ®**
www.editorialmarcaribe.es
Av. Gral. Flores 547, 70000 Col. del Sacramento, Department of Colonia, Uruguay.

Cover design and illustrations: *Isbelia Salazar Morote*

E-book available at:
https://editorialmarcaribe.es/ark:/10951/isbn.9789915698397

Format: Electronic

ISBN: 978-9915-698-39-7

ARK: ark:/10951/isbn.9789915698397

**Editorial Mar Caribe**

# General methodological proposals for Computer Science research: Prototyping, mathematical modeling, and the scientific method

**Colonia, Uruguay**

**2025**

# General methodological proposals for Computer Science research: Prototyping, mathematical modeling, and the scientific method

# Index

# Index of tables

# Introduction

Research in Computer Science occupies, today, a unique place in the academic landscape. Unlike traditional natural sciences, which study pre-existing phenomena, or pure mathematics, which operates in a universe of logical abstractions, computation is constantly moving between the artificial and the theoretical, between the construction of artifacts and the discovery of fundamental laws. This hybrid nature has historically generated a certain methodological fragmentation: Do we research to build more efficient systems? To prove theorems about complexity? Or to empirically validate the behavior of an algorithm in the real world?

The present book, *"General methodological proposals for Computer Science research: Prototyping, mathematical modeling, and the scientific method",* was born from the need to unify and formalize these approaches. Far from seeing these methodologies as watertight compartments, this work proposes an integrative vision where software engineering, mathematical rigor and empirical inquiry are intertwined to give solidity to the advancement of computational knowledge. The central structure of this text is based on three fundamental pillars that, although different in their execution, are complementary in the search for scientific answers:

- *Prototyping:* Often wrongly relegated to a mere development activity, prototyping is claimed here as a legitimate and necessary research tool. In a field where systems reach levels of complexity unmanageable for purely theoretical analysis, prototyping allows you to explore the design space, identify hidden constraints, and validate the technical feasibility of an idea. It is not just a matter of "making it work", but of using the constructed artifact as a probe to interrogate technological reality and refine research questions.

- *Mathematical Modeling:* If the prototype anchors us in practical reality, the mathematical model elevates us towards the abstraction necessary to generalize. This book examines how mathematical formalization allows researchers to predict behaviors, ensure security properties, and optimize resources before writing a single line of code. Modeling is not just an exercise in rigor; It is the language that allows real-world problems to be translated into computable structures, offering a framework for the deductive reasoning that is the basis of computational theory.
- *The Scientific Method:* We address the application of the classical scientific method—observation, hypothesis, experimentation, and analysis—in a digital context. As computing moves into areas such as artificial intelligence, complex networks, and large-scale distributed systems, software behavior often becomes stochastic or emergent. Here, empirical validation becomes critical. We explore how to design controlled experiments, how to collect meaningful data, and how to apply rigorous statistical analysis to confirm that a proposed solution really represents a significant advance over the state of the art.

The purpose of these pages is not to prescribe a single "recipe" for research, but to provide the reader—whether graduate student, academic, or industrial researcher—with a versatile methodological toolbox. In the following chapters, we will break down each approach, discussing its advantages, its limitations, and, most importantly, its points of convergence.

It will be shown that the most impactful research usually occurs at the intersection of these methods: when a mathematical model guides the creation of a prototype, and this, in turn, is subjected to the scrutiny of the scientific method to validate theoretical predictions. By mastering these three dimensions, the

Computer Science researcher not only becomes a better system builder, but a more complete scientist, capable of producing knowledge that is both theoretically sound and practically relevant.

We invite the reader to go through these methodological proposals with an open mind, willing to understand that in computing, theory and practice are not opposites, but the two indispensable sides of the same scientific coin.

# Chapter I.

# Methodological Pluralism in Computer Science: A Comprehensive Framework for Research Paradigms, Design Science, and Empirical Rigor

## 1. The Epistemological Crisis and the Search for Identity

Computer Science (CS) stands as a unique anomaly in the history of academic disciplines, vacillating between the abstract certainty of mathematics, the pragmatic utility of engineering, and the empirical observation of the natural sciences. This tripartite identity has engendered a rich, albeit often fragmented, landscape of methodological proposals. Unlike physics or biology, where the "Scientific Method" provides a relatively unified (though debated) foundation, computer science lacks a single, hegemonic research paradigm. Instead, it operates under a "methodological eclecticism" where the validity of a research contribution depends entirely on the ontological lens through which the computer program is viewed.[1]

The discipline has matured from its mid-20th-century roots—where it was often housed within departments of mathematics or electrical engineering—into a distinct field with its own rigorous standards. However, this maturation has not resolved the fundamental "ontological dispute" described by researchers at the University of South Carolina and others: Is a computer program a mathematical expression governed by deductive logic? Is it a technical artifact evaluated by its utility? Or is it a dynamic phenomenon, akin to a biological process, to be observed and measured?.[1]

These questions are not merely philosophical musings; they dictate the day-to-day mechanics of research. They determine whether a PhD student should spend their time proving a theorem, conducting a controlled user study, or building a prototype system. As the field expands into Artificial Intelligence (AI), Human-Computer Interaction (HCI), and massive distributed systems, the definitions of "rigor" are shifting. The rise of the "Replication Crisis" in AI and systems research has forced a re-evaluation of traditional publication models, leading to the adoption of pre-registered reports, artifact badging, and strict empirical standards.[5]

This report provides an exhaustive analysis of the methodological landscape of computer science. It dissects the three dominant paradigms—Rationalist, Technocratic, and Scientific—and explores the specific frameworks that operationalize these views, such as Design Science Research (DSR), Action Research, and the emerging protocols for Digital Twin validation and AI reproducibility. By synthesizing these diverse approaches, we aim to provide a unified guide for constructing rigorous, high-impact research in the computing disciplines.

# 2. The Tripartite Paradigms of Computing Research

To navigate the methodological options available, one must first locate their research within the broader epistemological traditions of the field. The literature consistently identifies three primary paradigms that govern how knowledge is acquired and validated in computer science.

## 2.1 The Rationalist Paradigm: Computing as Mathematics

The Rationalist paradigm posits that computer science is a branch of mathematics. In this view, the "computer program" is fundamentally a mathematical object—a formal expression that exists independent of any physical machine.[3] The ontology here is Platonist: algorithms and data structures are discovered, not invented.

**Methodological Implications:**

- **Epistemology:** Knowledge is *a priori*. It is derived through deductive reasoning from axioms, independent of sensory experience or physical experimentation.[1]
- **Core Method:** The primary method is the **Formal Proof**. Research involves defining a formal specification of a system and proving properties such as correctness, termination, and complexity bounds.[8]
- **Validation:** A result is valid if the proof is logically sound. Empirical testing is viewed with skepticism; as Dijkstra famously noted, testing can only show the presence of bugs, not their absence. Only formal verification provides certainty.[9]

Research in Theoretical Computer Science (TCS), semantics, and type theory operates almost exclusively within this paradigm. Here, the "methodology" section of a paper describes the formal system (e.g., lambda calculus, temporal logic) and the proof techniques (e.g., structural induction, diagonalization) employed.[10]

## 2.2 The Technocratic Paradigm: Computing as Engineering

The Technocratic paradigm views computer science as a design discipline, akin to chemical or aeronautical engineering. Here, the program is a "technical artifact"—a tool constructed to satisfy a specific set of requirements in a given context.[1] The ontology is functionalist: a program is defined by what it *does* and how well it solves a problem.

**Methodological Implications:**

- **Epistemology:** Knowledge is *probabilistic* and *pragmatic*. It is derived *a posteriori* through the construction and evaluation of artifacts.
- **Core Method:** The primary method is **Design Science Research (DSR)**. This involves an iterative cycle of problem identification, artifact construction, and evaluation.[12]
- **Validation:** A result is valid if the artifact is shown to be useful, reliable, and efficient relative to existing solutions. "Correctness" is less about mathematical truth and more about specification compliance and user satisfaction.[3]

This paradigm dominates Software Engineering (SE), Database Systems, and Network Architecture. The focus is on managing complexity and reliability in systems that are too large to be formally proven correct.[14]

### 2.3 The Scientific Paradigm: Computing as a Natural Science

The Scientific paradigm, arguably the most rapidly expanding view, treats computer systems as natural phenomena. This is particularly prevalent in Artificial Intelligence, Artificial Life, and complex large-scale networks. Here, the program is an empirical entity that exhibits behavior—often emergent and unpredictable—that must be observed.[1]

**Methodological Implications:**

- **Epistemology:** Knowledge is *empirical*. It is derived from observation, experimentation, and statistical inference.
- **Core Method:** The primary method is the **Controlled Experiment**. Researchers formulate hypotheses about system behavior and test them by manipulating independent variables (e.g., dataset size, network load) and measuring dependent variables (e.g., accuracy, latency).[9]
- **Validation:** A result is valid if it is statistically significant and reproducible. This paradigm embraces the "falsifiability" criterion of the natural sciences.[11]

This paradigm acknowledges that modern software systems—especially those involving deep learning or chaotic network dynamics—are often "black boxes" whose internal states are analytically intractable. Therefore, we must study them *in vivo* (in operation) rather than just *in vitro* (in static analysis).[16]

## 3. Design Science Research (DSR): The Engineering Standard

For researchers in Software Engineering and Information Systems who aim to build novel systems, Design Science Research (DSR) provides the codified methodological framework. Unlike routine design (which applies known solutions to known problems), DSR addresses "wicked problems" through the creation of innovative artifacts.[12]

## 3.1 The Peffers Process Model

The most widely cited framework for operationalizing DSR is the process model developed by Peffers et al. (2007). This model provides a nominal process for conducting and presenting design research, ensuring rigor in what might otherwise be ad-hoc development.[13] The process consists of six distinct steps:

1. Problem Identification and Motivation:
   The researcher must define the specific research problem and justify the value of a solution. This grounds the research in practical relevance. For example, "Current distributed ledgers cannot scale to global retail transaction volumes".13 The output of this phase is a definition of the problem space and the criteria for a successful solution.
2. Define Objectives for a Solution:
   Based on the problem definition, the researcher infers the objectives of a potential solution. These objectives can be quantitative (e.g., "Must process 10,000 transactions per second") or qualitative (e.g., "Must preserve user anonymity"). This step bridges the gap between the problem context and the technical architecture.13
3. Design and Development:
   This is the core creative step where the Artifact is created. In CS, an artifact can be a Construct (vocabulary/symbols), a Model (abstraction/representation), a Method (algorithm/practice), or an Instantiation (working system/prototype).13 The researcher determines the artifact's functionality and architecture.
4. Demonstration:
   The efficacy of the artifact is demonstrated by solving the problem in a suitable context. This could involve a simulation, a case study, a proof-of-concept implementation, or a pilot project. The goal is to show that the artifact can solve the problem, not necessarily

that it is the optimal solution yet.13

5. Evaluation:

   This is the critical differentiator between engineering and research. The researcher must observe and measure how well the artifact supports the solution. Evaluation methods vary by artifact type:

   ○ **Observational:** Case studies or field studies (e.g., deploying a new software process in a company).
   ○ **Analytical:** Static analysis, complexity analysis, or architectural review.
   ○ **Experimental:** Controlled experiments comparing the artifact to state-of-the-art baselines.
   ○ **Testing:** Functional (black-box) or structural (white-box) testing.
   ○ **Descriptive:** Using scenarios or walkthroughs to argue for utility.[13]

6. Communication:

   The problem, the artifact, and the evaluation are communicated to relevant audiences (researchers and practitioners). The structure of DSR papers typically follows this process logic.13

## 3.2 The Three Cycles of DSR Relevance

Hevner et al. expanded on this by defining DSR as a set of three cycles that ensure the research remains grounded [18]:

- **The Relevance Cycle:** Connects the research to the "Environment" (people, organizational systems, technical infrastructure). This provides the requirements and the testing ground for field testing.
- **The Design Cycle:** The central iteration of building and evaluating the artifact. This is where the hard technical work occurs.
- **The Rigor Cycle:** Connects the research to the "Knowledge Base" (scientific theories, existing methods, experience). This ensures the researcher draws on past work and contributes new knowledge back to the repository, preventing the "reinventing the

wheel" syndrome.[18]

### 3.3 Application in Sustainability and Requirements Engineering

Recent applications of DSR have demonstrated its utility in complex, value-driven domains. For instance, the development of the Sustainability Awareness Framework (SusAF) used DSR to create a tool that helps software engineers anticipate the long-term sustainability effects of their systems.[20] Similarly, researchers have used DSR cycles to integrate the United Nations Sustainable Development Goals (SDGs) into the Requirements Engineering process, showing that design artifacts can drive broad societal goals.[20] This highlights DSR's capacity to handle "wicked" sociotechnical problems where the requirements are not fully known at the outset.

## 4. Empirical Methodologies: The Scientific Method in Computing

As computer science moves away from pure theory, the adoption of the "Scientific Paradigm" has necessitated the adaptation of empirical methods from the natural and social sciences. This is most visible in the subfields of Human-Computer Interaction (HCI), Empirical Software Engineering (ESE), and High-Performance Computing (HPC).[9]

### 4.1 The Taxonomy of Experiments

Not all "experiments" in CS are the same. A nuanced understanding of experimental types is crucial for methodological clarity [9]:

- **Feasibility Experiments:** Often called "existence proofs," these demonstrate that a new tool or technique is possible. They are common in systems research (e.g., "We built a compiler that optimizes for X").
- **Trial Experiments:** Evaluating a single system to characterize its performance (e.g., "How does this algorithm scale with N?").

- **Comparison Experiments:** The gold standard in algorithmic research. Two or more systems are run under identical conditions to determine which performs better regarding specific metrics (time, space, accuracy).
- **Controlled Experiments:** Borrowed from psychology/physics, where independent variables are manipulated to measure their effect on dependent variables while holding confounding factors constant.

## 4.2 Challenges in Testing Scientific Software

Testing software used for scientific discovery (e.g., climate models, physics simulations) presents unique methodological challenges identified as the Oracle Problem. In traditional business software, the "correct" answer is known (e.g., 2+2=4). In scientific software, the correct answer is often the unknown value the software is being written to discover.[21]

- **Methodological Solutions:** Researchers must rely on Metamorphic Testing (checking if changes in input produce expected changes in output, even if the absolute output is unknown) and Code Clone Detection to ensure consistency.
- **Cultural Friction:** There is often a disconnect between domain scientists (who view the code and the model as inseparable) and software engineers (who view them as distinct). Effective methodology requires bridging this gap through rigorous documentation and unit testing of sub-components where the answer *is* known.[21]

## 4.3 Simulation as a Methodological Proxy

When physical experimentation is impossible—due to cost, scale (e.g., the Internet), or danger (e.g., nuclear simulations)—Simulation becomes the primary method. However, a simulation is only as good as its correspondence to reality.[22]

- **Verification:** The process of ensuring the computer program (the simulator) correctly implements the conceptual model. "Did we build the model right?" This involves standard software testing techniques.

- **Validation:** The process of ensuring the model accurately represents the real-world system. "Did we build the right model?"
    - **Retrodiction:** Validating the model by feeding it historical input data and checking if it reproduces the historical output data.[24]
    - **Prediction:** Validating by comparing model forecasts with future real-world events.
- **Sensitivity Analysis:** A critical methodological step where input parameters are systematically varied to determine how sensitive the output is to uncertainty. If small changes in input cause massive changes in output, the model may be too unstable for reliable conclusions.[25]

# 5. Methodologies in Artificial Intelligence and Machine Learning

The rapid ascent of AI has birthed its own set of methodological norms and crises. The complexity of modern deep neural networks (DNNs) means they act as "black boxes," requiring new investigative techniques to understand *why* they work.[26]

## 5.1 The Ablation Study: Isolating Causality

One of the most important methodological contributions from the AI community is the Ablation Study. In complex learning systems with many components (e.g., a new loss function, a specific layer type, a data augmentation technique), it is insufficient to show that the "whole package" works.

- **Mechanism:** An ablation study systematically removes one component at a time (e.g., "ablating" the attention mechanism) to measure its specific contribution to the overall performance.[26]
- **Purpose:** This serves as a "sensitivity analysis" for architectural decisions. It prevents "feature creep" where researchers add unnecessary complexity that does not actually

contribute to the result. In modern AI conferences (NeurIPS, ICML, ICLR), an ablation study is often a mandatory requirement for acceptance.[27]

### 5.2 The Crisis of Reproducibility and Benchmarking

AI research faces a severe reproducibility crisis. Models often perform well on specific benchmarks but fail to generalize.

- **Benchmarking Methodologies:** While benchmarks (like ImageNet or GLUE) provide a standard yardstick, over-reliance leads to overfitting the benchmark. Methodologically sound research now requires Out-of-Distribution (OOD) testing, where models are evaluated on data that differs statistically from the training set.[28]
- **The AI Scientist Concept:** New research is exploring the use of "AI Scientists"—automated agents powered by LLMs that can generate hypotheses, write code, run experiments, and even draft papers. While this promises to accelerate discovery, it introduces methodological risks regarding the verification of these automated findings. Benchmarks like **A2D** are being developed to test these agents on their ability to conduct reproducible science.[28]

### 5.3 Benchmarking for Reproducibility Agents

Recent initiatives are attempting to automate the reproducibility check itself. Benchmarks are being created to test whether AI agents can take a published paper's codebase and data and successfully reproduce the reported results. This represents a "meta-methodology"—using AI to verify the methodology of AI research.[27]

## 6. Action Research and Qualitative Inquiry

Not all computer science is quantitative. In fields like Information Systems and industrial Software Engineering, the human element is paramount. Here, **Action Research (AR)** and **Case Study** methodologies are essential.[16]

### 6.1 Action Research: Intervening to Learn

Action Research differs from traditional observational science in that the researcher actively participates in the system to improve it. It is highly relevant for introducing new software methodologies (e.g., Agile, DevOps) into an organization.[31]

- **The Cycle:** The process involves **Diagnosing** (identifying the problem in the organization), **Action Planning** (designing the intervention), **Action Taking** (implementing the change), **Evaluating** (measuring the impact), and **Specifying Learning** (deriving generalizable knowledge).[31]
- **Distinction from Consulting:** Unlike consulting, which focuses solely on solving the client's problem, AR focuses on generating scientific knowledge. The researcher must have a theoretical framework *before* the intervention and must reflect on the theoretical implications *after*.[33]

### 6.2 Ethnography and Grounded Theory

In Human-Computer Interaction (HCI), understanding the "user" requires more than surveys.

- **Ethnography:** Involves deep immersion in the user's environment to understand the social and contextual factors of technology use. This is crucial for "Contextual Design".[34]
- **Grounded Theory:** A bottom-up methodology where the researcher does not start with a hypothesis. Instead, they collect data (interviews, observations) and code it to allow a theory to "emerge" from the data. This is particularly useful for exploring new phenomena where no existing theory exists.[30]

## 7. Digital Twins: The Convergence of Modeling and Engineering

The concept of the Digital Twin (DT) represents a convergence of simulation, data science, and systems engineering. A Digital Twin is a dynamic virtual replica of a physical asset (e.g., a jet engine, a factory) that is continuously updated with real-time data.[37]

### 7.1 Validation methodologies for Digital Twins

Validating a DT is more complex than validating a static simulation because the physical system changes over time (degradation, maintenance).

- **TEVV (Testing, Evaluation, Verification, and Validation):** A comprehensive framework for DTs. It involves Unit Testing of individual models, Integration Testing of the data pipelines, and Performance Evaluation of the predictive accuracy.[37]
- **Data Provenance and Security:** Since DTs rely on real-time IoT data, the validation methodology must include checks for data integrity and anomaly detection. A "poisoned" data stream can invalidate the twin's predictions.[37]
- **Lifecycle Validation:** The validation is not a one-time event but a continuous loop. As the physical asset ages, the digital model must be re-calibrated and re-validated (a process known as "Model Updating" or "Continuous Validation") to ensure it remains a faithful representation.[38]

## 8. The Crisis of Rigor: Artifact Review and Empirical Standards

The increasing complexity of software and data has led to a recognition that a text-based research paper is an insufficient record of scientific work. "An article about computational science... is not the scholarship itself, it's merely scholarship advertisement. The actual scholarship is the complete software development environment and the complete set of instructions which generated the figures".[40]

### 8.1 ACM Artifact Review and Badging

To address this, the ACM and other organizations have instituted Artifact Review processes. Authors submit their code, data, and scripts alongside the paper.[41]

- **Badges:**
    - **Artifacts Available:** Code is permanently archived (e.g., on Zenodo or ACM DL).
    - **Artifacts Evaluated – Functional:** The artifact runs and produces the expected results.
    - **Artifacts Evaluated – Reusable:** The artifact is well-documented and can be easily reused by others.
    - **Results Reproduced:** An independent team used the author's artifacts to obtain the same results.
    - **Results Replicated:** An independent team obtained the same results using their *own* implementation.[42]
- **Impact:** This methodology forces researchers to prioritize "Practical Reproducibility" (e.g., using Docker containers) from day one, rather than trying to clean up code after acceptance.[44]

## 8.2 Registered Reports

To combat publication bias (where only positive results are published) and p-hacking (manipulating analysis to find significance), the field is adopting Registered Reports (RR).[6]

- **The Workflow:**
    1. **Stage 1:** Researchers submit the Introduction, Methodology, and Analysis Plan *before* collecting data.
    2. **In-Principle Acceptance (IPA):** If the methodology is sound, the journal commits to publishing the paper *regardless of the results*.[46]
    3. **Stage 2:** After data collection, the final paper is reviewed to ensure the protocol was followed.
- **Adoption:** First introduced in Software Engineering at the MSR 2020 conference, this format is now available in major journals like *Empirical Software Engineering* (EMSE) and *ACM Transactions on Information Systems* (TOIS).[6] It is particularly effective for

hypothesis-driven empirical studies.

### 8.3 ACM SIGSOFT Empirical Standards

To standardize the quality of reviews, the ACM SIGSOFT community has released Empirical Standards for common methodologies (Experiments, Case Studies, Benchmarking, etc.).[7]

- **Usage:** These standards provide checklists of "Essential," "Desirable," and "Extraordinary" attributes. Researchers use them to design studies, and reviewers use them to evaluate papers.
- **Benefit:** This reduces the subjectivity of peer review and provides clear guidance on what constitutes a "methodologically sound" contribution.[36]

## 9. Ethics as a Methodological Component

Ethics is no longer an afterthought; it is a core component of research methodology. The integration of "Society, Ethics, and Professionalism" (SEP) into the ACM/IEEE CS2023 curriculum highlights this shift.[51]

### 9.1 Institutional Review Boards (IRB)

For any research involving human subjects—which includes user studies, surveys of developers, and even analyzing data from public repositories if it contains PII (Personally Identifiable Information)—IRB approval is a mandatory methodological step.[53]

- **Risk Minimization:** The protocol must detail how risks to participants (including privacy risks) are minimized.
- **Informed Consent:** Researchers must document how participants are informed of the study's purpose and their rights.
- **Snowball Sampling:** Special care is needed for "snowball sampling" (participants recruiting others), as this can introduce coercion or reveal social network structures, requiring specific ethical safeguards.[54]

### 9.2 Value Sensitive Design (VSD)

VSD is a methodology that accounts for human values (privacy, autonomy, trust, environmental sustainability) throughout the design process.[55]

- **Methodology:** It involves Conceptual Investigations (identifying values), Empirical Investigations (understanding how stakeholders prioritize values), and Technical Investigations (designing the system to support these values).
- **Green Computing:** With the massive energy footprint of AI, "Green Technologies" and sustainability impact assessments are becoming standard methodological requirements for systems research.[20]

## 10. Structuring the Rigorous Research Proposal

The methodology of computer science is a composite discipline that draws on the rigor of mathematics, the creativity of engineering, and the empiricism of the natural sciences. A strong research proposal must clearly articulate its paradigm and adhere to the specific standards of that tradition (see Table 1).

### Table 1: Summary of Methodological Frameworks by Research Goal

| Research Goal | Primary Paradigm | Key Methodology | Validation Standard |
|---|---|---|---|
| **Proving a Theorem** | Rationalist | Deductive Reasoning, Proof Theory | Mathematical Correctness |
| **Building a System** | Technocratic | Design Science Research (DSR) | Utility, Efficiency, Relevance |
| **Understanding a Phenomenon** | Scientific | Controlled Experiment, Simulation | Statistical Significance, Reproducibility |

| | | | |
|---|---|---|---|
| **Improving a Process** | Sociotechnical | Action Research, Case Study | Organizational Learning, Adoption |
| **Creating an AI Model** | Scientific/Engineering | Ablation Study, Benchmarking | State-of-the-Art Performance, Generalizability |

Whether one is proving the properties of a cryptographic protocol, designing a sustainable software architecture, or conducting a large-scale randomized control trial on user interface design, the unifying theme is rigor. By adopting formal frameworks like Peffers' DSR model, adhering to Artifact Review standards, and pre-registering empirical hypotheses, computer scientists can ensure their work stands on a solid methodological foundation, capable of weathering the scrutiny of peer review and contributing lasting value to the body of knowledge. The future of the field belongs to the "methodological polyglot"—the researcher who can fluidly move between the whiteboard proof, the git repository, and the statistical analysis package.

# Chapter bibliography


1. Abla, D. E. (2015). Agro-Input Use in Peri-Urban Production in the Greater Accra Region. *(Doctoral Dissertation, University of Ghana).*
2. Alberto, S. P. C. (2015). "USO DE LA TECNOLOGÍA E INFORMÁTICA COMO FACTOR DETERMINANTE EN LA CALIDAD ESTUDIANTIL DEL NIVEL MEDIO EN EL INSTITUTO JACOBO ARBENZ GUZMÁN, QUETZALTENANGO". *World Agriculture*, *1*(May).
3. Almeida, C. S. de, Miccoli, L. S., Andhini, N. F., Aranha, S., Oliveira, L. C. de, Artigo, C. E., Em, A. A. R., Em, A. A. R., Bachman, L., Chick, K., & Curtis, D. (2018). Republic of Kenya, Ministry of Environment , Water and Natural Resources Department of Environment and Natural Resources, Policy Draft April 2017. *Universty of South*

*Africa.*, *5*(4).

4. Andrade, F., Correa, A., García, N., Rodilla, V., Villagrasa, V., & Nuria Linares, G. (2020). Plantas Medicinales. In *Arts in Psychotherapy* (Vol. 291, Issue April).
5. Apriliyani. (2015). Sistem Penentuan Tingkat Resiko Penyakit Jantung Koroner Dan Kardiovaskuler Menggunakan Metode Framingham Score. *World Agriculture*, *1*(May).
6. ARIAS ÁLVAREZ, M. C. (2010). SUPERHEATED STEAM GENERATION IN A FRESNEL SOLAR FIELD FOR HYBRIDIZATION OF A COAL POWER PLANT. *World Agriculture*, *1*(May).
7. Aspling, F. (2020). Unleashing Animal-Computer Interaction: A Theoretical Investigation of the "I" in ACI. In *Report Series / Department of Computer & Systems Sciences* (Issue 20).
8. Bastedo, M. N., & Bowman, N. A. (2015). Improving Admission of Low-SES Students at Selective Colleges: Results from a Natioanl Experiment. *Educational Researcher*, *33*.
9. BORGHI, C. M. S. D. O. (2015). DETERMINAÇÃO SOCIAL DO PROCESSO SAÚDE-DOENÇA: QUE VOZES ECOAM NO BRASIL? *World Agriculture*, *1*(May).
10. Briones, M. (2015). Diseño e Implementación de un Plan Estratégico de Comunicación para Fortalecer los Mecanismos de Comunicación e Identidad de las Empresas Comercializadoras de Ropa Deportiva, caso Marathon Sports. *UTE*, *1*(May).
11. César Augusto, V. V. (2015). La Curva Hibrida De Riesgo: Análisis Retrospectivo Y Prospectivo Del Riesgo Por Fenómenos Naturales. *Tesis*, *1*(May).
12. Connop, S., Vandergert, P., Eisenberg, B., Collier, M. J., Nash, C. C., Clough, J., Newport, D. J., Cameron, R. W. F., Blanuša, T., McClymont, K., Hansen, R., Pauleit, S., Marsboom, C., Vrebos, D., Staes, J., Meire, P., Wang, J., Banzhaf, E., Iojă, C. I., … McLean, N. (2017). Sustainability. *Sustainability*, *9*(1).
13. Courbet, D. (2013). Pour un ddveloppement raisonnn des mmthodes exppimentales en SIC. Quelques inttrrts pisttmologiques du pluralisme mmthodologique (For a Reasoned Development of Experimental Methods in Information and Communication Sciences: Some Epistemological Findings of Methodological Pluralism). *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.2314403

14. Courbet D. (2013). Pour un développement raisonné des méthodes expérimentales en SIC. Quelques intérêts épistémologiques du pluralisme méthodologique. *ESSACHESS - Journal for Communication Studies*, *6*(1).
15. Darusman, B. (2015). Persepsi Mahasiswa Terhadap Kontribusi Hmp Pgsd Dalam Mengoptimalkan Kemampuan Berpikir Kritis Mahasiswa Pgsd Fkip Universitas Muhammadiyah Surakarta. In *Universitas Muhammadiyah Surakarta* (Vol. 1, Issue May).
16. Duad, A. M. (2015). Karakteristik Fisisk Daging Sapi Bali Pascarigor Yang Dimarinasi Theobromin Pada Level Dan Lama Marinasi Yang Berbeda. *World Agriculture*, *1*(May).
17. Garzón, Y. (2015). ARTETERAPIA COGNITIVA CONDUCTUAL PARA EL TRATAMIENTO DE ANSIEDAD Y DEPRESIÓN EN UN GRUPO DE PACIENTES ONCOLÓGICOS DE LA ASOCIACIÓN NACIONAL CONTRA EL CÁNCER CAPÍTULO DE VERAGUAS. *World Agriculture*, *1*(May).
18. Giacomarra, E. J. (2015). Heritage Education for School-Aged Children: an Analysis of Program in Salem, Massachusetts. *World Agriculture*, *1*(May).
19. Gomes, Í. de A. (2015). Desenvolvimento Experimental de um Secador Solar de Frutas com Aproveitamento Multienergético. In *Climate Change 2013 - The Physical Science Basis* (Vol. 1, Issue May).
20. Goncalves Dos Santos, K. C. (2015). Interaction of Ilex paraguariensis with beneficial microorganisms. *Nhk技研*, *151*(1).
21. Gordon, J. S. (2020). Building Moral Robots: Ethical Pitfalls and Challenges. *Science and Engineering Ethics*, *26*(1). https://doi.org/10.1007/s11948-019-00084-5
22. Graha, arya bintang, Ginting, M., & Tobing, E. (2015). ANALISA PRESSURE BUILD UP DAN INTERFERENCE TEST PADA SUMUR ALPHA DAN "BETA LAPANGAN X. *Seminar Nasional Cendekiawan*, *1*(May). https://doi.org/10.1017/CBO9781107415324.004
23. Grams, D., Farrell, B., Mackintosh, J. D., Berry, C., Liscutin, N., Aydinlioğlu, Ö., Broadhead, S., Puccio, G., Ravnborg, J. L., Monaghan, S., Bursk, E. C., Sethi, B. S.,

Dunbar, D. S., Bergh, B. G. v, Reid, L. N., Schorin, G. A., Roberts, J., Roberts, T., Bergh, B. G. v, … Meron, Y. (2024). Scopus Document. *Journal of Advertising*, *12*(1).

24. Holeman, I. (2018). Room for Silence: Ebola Research, Pluralism and the Pragmatic Study of Sociomaterial Practices. *Computer Supported Cooperative Work: CSCW: An International Journal*, *27*(3–6). https://doi.org/10.1007/s10606-018-9329-x
25. Hopkins, K., Mutasam, U., Ibrohim, I., Susilo, H., Puttick, S., Cullinane, A., Voss, S., Kruse, J. W., Kent-Schneider, I., Wei, B., Jiang, Z., Gai, L., Torres, J., Vasconcelos, C., Cavallo, A. A., Burgers, H., Ghezzi, A., van de Vrande, V., Erduran, S., … Schmidt Kjærgaard, R. (2020). Exploring the Use of Lesson Study to Develop Elementary Preservice Teachers' Pedagogical Content Knowledge for Teaching Nature of Science. *Science and Education*, *16*(2).
26. Interculturales, C. de E. M. (2020). Manual para la promoción del buen cultivo y uso de plantas medicinales. In *Arts in Psychotherapy*.
27. Jakub Kolacz. (2015). ENERGY MINIMIZATION IN NEMATIC LIQUID CRYSTAL SYSTEMS DRIVEN BY GEOMETRIC CONFINEMENT AND TEMPERATURE GRADIENTS WITH APPLICATIONS IN COLLOIDAL SYSTEMS. *World Agriculture*, *1*(May).
28. Jung, E. E. J., Jung, E. E. J., Ye, J., Kurashima, Y., Kobayashi, T., Tsuda, H., Takahara, T., Sakurai, W., Lima, L. M. M. de, Sá, L. R. de, Macambira, A. F. U. dos S., Nogueira, J. de A., Vianna, R. P. de T., Moraes, R. M. de, Barzegar, M., Sadeghi-Niaraki, A., Shakeri, M., Choi, S.-M., Oduntan, G., … Buchanan, R. (2020). Abstracts. *Journal of Planning Literature*, *10*(1).
29. Kaplan, B. (2001). Evaluating informatics applications - Some alternative approaches: Theory, social interactionism, and call for methodological pluralism. *International Journal of Medical Informatics*, *64*(1). https://doi.org/10.1016/S1386-5056(01)00184-8
30. Kasza, G. (2001). Perestroika: For An Ecumenical Science of Politics. *PS: Political Science and Politics*, *34*(3).
31. Levi-Faur, D. (2006). A question of size? A heuristics for stepwise comparative research design. In *Innovative Comparative Methods for Policy Analysis: Beyond the*

*Quantitative-Qualitative Divide*. https://doi.org/10.1007/0-387-28829-5_3

32. Linares, N. (2013). Plantas Medicinales.Farmacia de la naturaleza. *UPA MADRID*, *291*(April).
33. Linares, N. (2020). Plantas Medicinales, Cuaderno de trabajo. *Arts in Psychotherapy*, *291*(April).
34. Lubis, M. Z. (2015). Tingkat Kesukaan dan Daya Terima Makanan serta hubungannya ddengan Kecukupan Energi dan Zat Gizi pada Santri Putri MTs Darul Muttaqien Bogor. *World Agriculture*, *1*(May).
35. Lund, S. (2015). THE COMEDY OF FRENCHNESS: EXPLORING SOCIAL ISSUES IN MAINSTREAM FRENCH CINEMA. *World Agriculture*, *1*(May).
36. MANUEL, V. M. J. (2015). HERRAMIENTAS FINANCIERAS PARA LA COMPAÑÍA "AMARY´S", DE LA CIUDAD DE QUITO PROVINCIA DE PICHINCHA. *UNIVERSIDAD REGIONAL AUTÓNOMA DE LOS ANDES "UNIANDES - IBARRA,"* *1*(May).
37. McCloskey, D. (1999). Other Things Equal - Economical Writing: An Executive Summary. *Eastern Economic Journal*, *25*(2).
38. Nantwi2, S. K. N. and W. K. (2013). British Journal of Education, Learning and Development Psychology. *Jurnal EMBA*, *1*(4).
39. Nisa, A. N. (2015). ANALISIS USER EXPERIENCEMEDIA SOSIAL MINDTALK. *World Agriculture*, *1*(May).
40. Núñez, G. (2015). Dimensiones de personalidad y estilos de afrontamiento en pacientes con diagnóstico de cáncer. *World Agriculture*, *1*(May).
41. Pardo, L. (2015). ELABORACIÓN DEL MAPA TENENCIAL Y PRELIMINAR CATASTRAL DE LA PROVINCIA DE CHIRIQUI PARA EL PROYECTO DE REGULARIZACION DE TIERRAS DE LA REPÚBLICA DE PANAMÁ. *World Agriculture*, *1*(May).
42. Rosenberg, M. (2016a). Health geography II. *Progress in Human Geography*, *40*(4). https://doi.org/10.1177/0309132515581094
43. Rosenberg, M. (2016b). Health geography II: 'Dividing' health geography. *Progress in*

*Human Geography*, *40*(4). https://doi.org/10.1177/0309132515581094

44. Stobbe, G. (2013). Just Enough ENGLISH GRAMMAR. In *Government Information Quarterly* (Vol. 4, Issue 1).
45. Sugiarti. (2015). Hubungan antara Kondisi Perairan dan Hasil Tangkapan Ikan di Muara - Muara Sungai Di Teluk Banten. *Institut Pertanian Bogor*.
46. Sumaryani, S., Manangin, yunita restiasa, & Astuti, S. (2015). senam nifas pada ibu post partum di desa meijing,yogyakarta. In *World Agriculture* (Vol. 1, Issue May).
47. Swanson, T. (2015). Consensus-as-a-service: a brief report on the emergence of permissioned, distributed ledger systems. Work. *World Agriculture*, *1*(May).
48. Tiara, R. (2015). Pengaruh Relationship Marketing Terhadap Customer Retention (Retensi Pelanggan) Pada Wardah Beauty House Yogyakarta. In *World Agriculture* (Vol. 1, Issue May).
49. Turobov, A., Chumakova, M., & Vecherin, A. (2019a). World best practices in applying mathematical and statistical crime prediction algorithms. *Mezhdunarodnye Protsessy*, *17*(4). https://doi.org/10.17994/IT.2019.17.4.59.9
50. Turobov, A., Chumakova, M., & Vecherin, A. (2019b). World best practices in applying mathematical and statistical crime prediction algorithms; [МЕЖДУНАРОДНЫЙ ОПЫТ ПРИМЕНЕНИЯ МАТЕМАТИКОСТАТИСТИЧЕСКИХ АЛГОРИТМОВ ПРОГНОЗИРОВАНИЯ ПРЕСТУПНОСТИ]. *Mezhdunarodnye Protsessy*, *17*(4).
51. Univ Lampung. (2015). Metode Pengukuran Rendemen Rumput Laut. *Skripsi*, *1*(May).
52. VELOSO, F., BARBOSA FILHO, F. D. H., & PERUCHETTI, P. (2022). Impactos da educação no mercado de trabalho. *Instituto Brasileiro de Economia*.
53. VILLENA, C. (2015). DISEÑO DE UN PLAN DE MARKETING ESTRATÉGICO PARA LA ESTACIÓN DE SERVICIOS VIGUESAM. *World Agriculture*, *1*(May).
54. Равочкин, Н. Н. (2021). SOCIAL IDEAS DISTRIBUTION IN CONTEMPORARY SOCIETY. *Вестник Тверского Государственного Университета. Серия: Философия*, *1(55)*. https://doi.org/10.26456/vtphilos/2021.1.031

55. Туробов Алексей, Чумакова Мария, & Вечерин Александр. (2019). Международный опыт применения математико статистических алгоритмов прогнозирования преступности. *Международные Процессы*, *17*(4).

# Chapter II.

# Methodological Foundations of Computer Science: A Comprehensive Analysis of Prototyping, Mathematical Modeling, and the Scientific Method

## 1. The Epistemological Pluralism of Computer Science

Computer Science (CS) occupies a unique and often contested epistemological space, straddling the boundaries between the abstract precision of mathematics, the pragmatic constructivism of engineering, and the observational rigor of the natural sciences.[1] Unlike traditional disciplines defined by a singular mode of inquiry—such as the deductive proofs of mathematics or the inductive experiments of biology—computer science is intrinsically pluralistic. It investigates phenomena that are at once artificial (created by humans) and mathematical (governed by formal logic), necessitating a research methodology that is robust enough to handle this duality.[2]

The discipline has historically struggled with an "identity crisis," oscillating between being viewed as a field deeply rooted in strong theories—such as computational complexity, Turing machines, and formal semantics—and an engineering discipline focused on the creation of artifacts that transform society, such as the Von Neumann architecture, the internet, and distributed systems.[1] This duality suggests that computer science inherits its research methods from both ancestors: the mathematical approach utilizing axioms, postulates, and proofs; and the engineering approach employing quantification, measurements, and comparison.[1] Furthermore, as computer systems have grown to levels of complexity where their behavior cannot always be predicted solely through analysis, researchers increasingly treat them as natural objects to be observed, measured, and subjected to hypothesis testing, introducing the empirical or scientific paradigm.[2]

Consequently, the methodological landscape of modern computer science can be broadly categorized into three primaries, yet interconnected, proposals: Mathematical Modeling, Prototyping (often framed within Constructive Research or Design Science), and the Scientific Method.[5] Each methodology offers unique advantages and is suited for different types of research inquiries. Mathematical modeling provides a framework for abstract reasoning, enabling researchers to simulate systems and predict their behaviors through formal logic.[5] Prototyping enables researchers to create tangible representations of their ideas, facilitating iterative testing, refinement, and the demonstration of feasibility.[5] Meanwhile, the scientific method provides a structured approach to inquiry, emphasizing the systematic testing of hypotheses and the validation of empirical findings through controlled experimentation.[5]

This report provides an exhaustive analysis of these three methodological pillars. It explores not only their individual mechanics and validity criteria but also the "interplay" among them that enriches the research landscape.[5] By synthesizing theoretical foundations with practical applications, this analysis elucidates how computer science generates knowledge, bridging the gap between the abstract "universe" of the computer—an ever-developing artifact—and the concrete reality of its application in society.[2]

## 1.1 The "Sciences of the Artificial"

To understand the methodologies of computer science, one must first define the object of investigation. In physics or biology, the object of study usually pre-exists the observer. In computer science, the universe of study is the computer itself—an artifact created by human intelligence.[2] This aligns computer science with Herbert Simon's concept of the "Sciences of the Artificial," where the goal is not merely to describe the world as it is, but to design courses of action to change existing situations into preferred ones.[7]

This artificial nature complicates the application of traditional scientific methods. If the object of study is a program written by the researcher, "experimentation" takes on a different meaning than in the natural sciences. It may refer to "demonstration" (showing that the artifact functions as intended) rather than "hypothesis testing" (proving a universal theory true or false).[3] This distinction is critical in separating **constructive research**, which aims to build a solution to a specific persisting problem, from **empirical research**, which aims to test the feasibility or performance of that solution using empirical evidence.[8]

# 2. Mathematical Modeling: The Theoretical Pillar

Mathematical modeling serves as the theoretical backbone of computer science, providing the necessary tools for abstraction, specification, and verification. In this paradigm, computer science is treated as a branch of applied mathematics, where systems are specified using logic, probability, and algebra, and properties are verified through deductive reasoning rather than observation.[2]

## 2.1 The Role of Abstraction and Formalism

Abstraction is the process of developing a conceptual veneer that hides the complexity of internals, allowing researchers to focus on the essence of a problem without being bogged down by implementation details.[9] Mathematical modeling makes this abstraction precise. By stripping away extraneous variables, researchers can create a model that captures the fundamental behavior of a system, whether it is an algorithm, a network protocol, or a database transaction system.[2]

This reliance on abstraction aligns computer science with the "Analytic Paradigm," where researchers propose formal theories or sets of axioms and derive results that are then compared with empirical observations.[10] The primary tool here is Formal Methods, which rely on abstract models to represent system properties mathematically and precise semantics to define the meaning of these models without ambiguity.[11]

### 2.1.1 Taxonomy of Formal Approaches

Formal methods are broadly categorized into two approaches based on their focus:

- **Model-Oriented Formalisms**: These focus on constructing explicit mathematical models of the system's state and operations. They allow for detailed simulation and refinement, making them suitable for constructive design. Examples include Z, VDM, and state transition systems.[12]
- **Property-Oriented Formalisms**: These emphasize axiomatic descriptions of desired behaviors and invariants. They use logical predicates to assert *what* the system must satisfy without prescribing *how*. This distinction influences their applicability: model-oriented suits constructive design, while property-oriented excels in abstract validation.[12]

## 2.2 The Research Cycle in Theoretical Computer Science (TCS)

Research in Theoretical Computer Science (TCS) follows a distinct lifecycle that mirrors the mathematical tradition rather than the empirical scientific method. It is characterized by a "Formal Methodology," where the basic idea is to abstract away as many details as possible to leave behind only the essence of the problem.[13]

The TCS research cycle typically involves the following phases:

1. **Formalization**: Converting a real-world problem into a mathematical object (e.g., a graph, a logic formula, or a state machine).[15] This step is critical because any error in formalization invalidates the subsequent proofs.
2. **Deduction and Proof**: Using axioms and proof techniques to derive properties of the model. Common techniques include inductive proofs, reduction (mapping a problem to a known solved/unsolved problem), and diagonalization.[1]
3. **Verification**: Proving that an algorithm or system design meets its specification for *all* possible inputs. This offers a guarantee of correctness that empirical testing can never provide, as testing can only show the presence of errors, not their absence.[16]

This methodology is most frequently used to prove facts about algorithms (e.g., time/space complexity) and systems (e.g., safety, liveness).[14] It underpins fields such as computational complexity theory, cryptography, and programming language theory.[1]

## 2.3 Formal Methods vs. Empirical Testing: A Fundamental Tension

A fundamental tension exists in computer science between formal methods and empirical testing. Formal methods offer the promise of *absolute* correctness—proving a system is error-free—whereas testing is inherently limited by the finite number of test cases that can be run.[16]

However, formal methods face significant challenges:

- **The "Gap" Problem**: A mathematical proof applies to the *model*, not the *implementation*. A recent trend in mathematical modeling is to publish computer code together with research findings, but this raises the question of whether the code faithfully implements the model.[18] Implicit assumptions in the code (e.g., regarding memory limits or floating-point precision) may not be reflected in the formal model, severing the causal link between the proof and the actual system behavior.[18]
- **Scalability**: Formal verification is often computationally expensive and difficult to scale to large, complex software systems. This has led to the development of "lightweight" formal methods and their integration with automated tools like model checkers.[12]
- **Learning Curve**: The mathematical disciplines used to formally describe computational systems (e.g., discrete mathematics, logic, automata theory) are often outside the domain of traditional engineering education, creating a barrier to adoption.[19]

Despite these challenges, the "Analytical Paradigm" remains essential. In critical systems—such as avionics, nuclear power, and medical devices—no serious engineer would build without mathematical modeling because of the risks involved. The cost of correcting design errors after implementation is prohibitive, making the predictive power of mathematical analysis invaluable for both safety and efficiency.[16]

### 2.4 Deterministic vs. Stochastic Modeling

Mathematical modeling in CS is not monolithic; it varies based on the nature of the system being studied (see Table 2).[21]

### Table 2: Deterministic vs. Stochastic Modeling

| **Modeling Type** | **Application Domain** | **Methodology** |
|---|---|---|
| **Deterministic Modeling** | Algorithm Analysis, Formal Verification, Logic | Uses deductive logic to prove properties that hold true under all conditions (e.g., sorting algorithm correctness). [21] |
| **Stochastic Modeling** | Networks, Performance Evaluation, Finance | Uses probability theory and statistics to model systems with inherent uncertainty or randomness (e.g., packet arrival rates, stock prices). [21] |

Stochastic modeling is particularly important in systems research, where researchers must account for variable workloads and environmental noise. Here, the mathematical model (e.g., a queuing theory model) provides a theoretical basis for extrapolation, allowing predictions about system behavior under loads that cannot be easily tested empirically.[4]

## 3. Prototyping and Constructive Research: The Engineering Pillar

While mathematical modeling dominates the theoretical side of CS, Prototyping and Constructive Research dominate the engineering and applied sides. This methodology is concerned with "building" as a primary research activity. It involves the construction of artifacts—software, hardware, methods, conceptual frameworks, or tools—to demonstrate feasibility, solve practical problems, or generate new knowledge through the act of creation.[14]

## 3.1 Constructive Research Methodology

Constructive research is a research procedure for producing innovative constructions, intended to solve problems faced in the real world and, by that means, to make a contribution to the theory of the discipline in which it is applied.[22] It differs from "scientific method" research, which aims to explain *why* something happens, and from pure "consulting," which aims to solve a problem without necessarily generating new theoretical knowledge.[22]

The constructive approach implies building an artifact (practical, theoretical, or both) that solves a domain-specific problem in order to create knowledge about *how* the problem can be solved in principle.[23] This methodology gives results which can have both practical and theoretical relevance, solving knowledge problems concerning feasibility, improvement, and novelty.[23]

### 3.1.1 The Constructive Research Cycle

The process of constructive research is often described as a multi-phase cycle. According to [24] and [22], the key phases are:

1. **Problem Identification**: Find a practically relevant problem that also has research potential. The problem must be significant enough to warrant a research effort.[22]
2. **Pre-understanding**: Obtain a general and comprehensive understanding of the topic. This involves literature reviews and studying existing solutions to ensure the new artifact will be novel.[24]
3. **Innovation (Construction)**: Construct the solution idea. This is the creative phase where the researcher designs the artifact (algorithm, system, model).[24]

4. **Demonstration**: Demonstrate that the solution works. This is often achieved through a "Proof of Concept" or a prototype implementation.[24]
5. **Theoretical Contribution**: Show the theoretical relevance of the construct. The researcher must articulate *why* the solution works and linking it back to the body of knowledge.[22]
6. **Validation/Applicability**: Examine the scope of applicability and generalize the findings. This often involves testing the artifact in a real-world or simulated environment.[22]

## 3.2 Design Science Research (DSR)

Closely related to constructive research is Design Science Research (DSR), a methodology prevalent in Information Systems and Software Engineering. DSR focuses on the creation and evaluation of IT artifacts intended to solve identified organizational problems.[25] DSR distinguishes itself by explicitly requiring a contribution to the "knowledge base" while solving an "environment" problem. DSR operates through three distinct cycles that ensure the research remains grounded and rigorous [22]:

- **The Relevance Cycle**: Connects the research to the environment (people, systems, organizations). It defines the problem requirements and provides the context for field testing.
- **The Design Cycle**: The core iterative process of building, testing, and refining the artifact. This is where prototyping takes place.
- **The Rigor Cycle**: Connects the research to the knowledge base (foundations, methodologies, past theories). It ensures the design is not just a "hack" but is grounded in existing scientific knowledge and contributes new knowledge back to the field.

### 3.2.1 Validity in DSR

A major challenge in DSR is ensuring the validity of the resulting artifacts. Artifacts are often criticized for lacking rigorous evaluation. To address this, DSR frameworks propose five essential validity types [26]:

1. **Instrument Validity**: Does the artifact function correctly? (Technical correctness).
2. **Technical Validity**: Does the artifact solve the technical problem it claims to?
3. **Design Validity**: Is the design structure sound and justifiable?
4. **Purpose Validity**: Does the artifact actually help the user or solve the real-world problem? (Utility).
5. **Generalization**: Can the artifact or its underlying principles be applied to other contexts?

Crucially, "instrument validity" and "design validity" are often the least developed in research papers, posing a risk of overlooked flaws that threaten research credibility.[26]

## 3.3 The Role of Prototyping

Prototyping is the central activity within the constructive and DSR methodologies. A prototype is an early sample, model, or release of a product built to test a concept or process.[28] It serves as a tangible representation of an idea, allowing for iterative testing and refinement.[5]

### 3.3.1 Research vs. Industrial Prototypes

It is vital to distinguish between Research Prototypes and Industrial Prototypes, as they serve different goals and are evaluated by different criteria (see Table 3).

## Table 3: Research vs. Industrial Prototypes

| **Feature** | **Research Prototype** | **Industrial (Commercial) Prototype** |
|---|---|---|
| **Goal** | Knowledge production; Proof of concept; Feasibility [29] | Product design; Usability; Manufacturability; Market viability [28] |

| | | |
|---|---|---|
| **Nature** | "Good enough" to serve as a vehicle for knowledge; Often uses "shortcut technology" or Wizard-of-Oz techniques [29] | Robust; Reflects final product; Focus on user experience and scale [28] |
| **Lifespan** | Short-lived; Often discarded after the concept is proven [29] | Iterative; Evolves into the final product or beta release [28] |
| **Evaluation** | Novelty; Theoretical contribution; Feasibility [22] | Reliability; Return on investment; Market demand; User satisfaction [30] |
| **User Role** | Participants in an experiment or demonstration [7] | Beta testers or potential customers [28] |

Research prototypes are not "deficient" versions of industrial prototypes; they are purpose-built instruments for inquiry.[29] Their quality is secondary to their ability to generate insight. For example, a research prototype might use a clumsy interface if the research question is about the underlying algorithm, whereas an industrial prototype would fail if the interface were poor.[29]

### 3.3.2 Proof of Concept (PoC)

A Proof of Concept (PoC) is an advanced form of prototyping often used to "prove" that a new technology, service, or idea is viable.[30] In computing literature, "experiment" is sometimes used synonymously with "demonstration" or "PoC".[3] A PoC differs from a standard prototype in that it may not be a working model of the entire system but rather a focused experiment to validate a specific critical function or claim.[30] It is the "existence proof" of the engineering world—demonstrating that a system with certain properties *can* be built.[14]

# 4. The Scientific Method: The Empirical Pillar

The third pillar, the Scientific Method, represents the empirical branch of computer science. While CS involves artificial objects, the behavior of complex software systems, networks, and human-computer interactions is often too complex to model analytically or predict solely through construction. Thus, researchers must observe the world, propose models/theories, measure, analyze, and validate hypotheses.[4]

## 4.1 Adapting the Scientific Method to the Artificial

The scientific method in CS follows the classical hypothetico-deductive pattern but with domain-specific adaptations.[32] The general cycle includes:

1. **Observation**: Noticing a phenomenon (e.g., "users struggle with this interface" or "this network protocol slows down under load").[33]
2. **Hypothesis Formulation**: Proposing a testable explanation or prediction. A hypothesis must be falsifiable (e.g., "Algorithm A is faster than Algorithm B for datasets of type X").[34]
3. **Experimentation**: Designing a controlled test (benchmarking, randomized control trial) to gather data. This involves identifying independent variables (what is changed), dependent variables (what is measured), and nuisance variables (what must be controlled).[35]
4. **Analysis**: Using statistical methods to accept or reject the hypothesis. This often involves Null Hypothesis Significance Testing (NHST).[34]
5. **Conclusion/Reporting**: Publishing results to contribute to the body of knowledge.[33]

In computer science, this method is applied across several sub-disciplines, each with its own norms and "experimental cultures".[3]

## 4.2 Empirical Software Engineering (ESE)

Empirical Software Engineering (ESE) applies the scientific method to the software development process itself. It aims to move the field from "advocacy research" (where researchers propose a method and claim it is good based on persuasion) to "evaluation research" (where claims are tested).[4]

### 4.2.1 ESE Strategies

ESE employs a variety of empirical strategies, broadly categorized into fixed and flexible designs [10]:

- **Controlled Experiments**: Performed in laboratory settings (e.g., using students as subjects) to test specific hypotheses with high internal validity. However, they often suffer from low external validity because the setting is artificial and the tasks are small.[10]
- **Case Studies**: In-depth inquiries into real-world projects. These have high external validity because they study professionals in their natural environment, but they are hard to control and generalize.[10]
- **Surveys**: Collecting qualitative or quantitative data from a broad population to understand trends, opinions, or adoption rates.[10]
- **Action Research**: A collaborative approach where the researcher actively participates in the development process to solve a problem while studying it. This blends constructive and empirical methods.[10]

### 4.2.2 The "ABC" Framework

To navigate the trade-offs in ESE strategies, researchers use the ABC Framework, which identifies three desirable aspects of research that cannot be maximized simultaneously [36]:

1. **A (Actors)**: Generalizability over the population (e.g., all software engineers).
2. **B (Behavior)**: Precision of control over the behavior (e.g., controlling the exact task duration).
3. **C (Context)**: Realism of the setting (e.g., a real company under deadline pressure).

A lab experiment maximizes B but sacrifices C and often A. A field study maximizes C but sacrifices B. A survey maximizes A but sacrifices B and C (since it relies on self-reporting). This framework helps researchers choose the right strategy for their specific research question.

## 4.3 Human-Computer Interaction (HCI)

HCI research relies heavily on the scientific method, borrowing techniques from psychology and social sciences to study how humans interact with technology.[34]

HCI studies are often categorized as:

- **Formative Evaluation**: Exploratory tests done during the design process to identify usability problems and refine the design. These are often qualitative and iterative.[38]
- **Summative Evaluation**: Controlled experiments done after the design is complete to measure performance (e.g., time on task, error rate) against a benchmark or a competitor. These are quantitative and hypothesis-driven.[38]

In HCI, the "scientific method" is often synonymous with Null Hypothesis Significance Testing (NHST). Researchers set up a null hypothesis ($H_0$: "there is no difference between Interface A and B") and try to reject it using statistical tests (t-tests, ANOVA) to show that a new design is significantly better.[34] However, the field is increasingly embracing qualitative methods (ethnography, grounded theory) to capture the richness of user experience that numbers alone cannot describe.[37]

## 4.4 Systems Research: The "Experimental" Debate

In the sub-fields of Computer Systems (Operating Systems, Networks, Databases), "experimentation" often refers to performance evaluation. However, the methodological rigor of this work has been a subject of intense debate.

### 4.4.1 The Rejuvenation of Experimental CS

In the 1980s and 90s, reports by Feldman and Sutherland critiqued the field for undervaluing experimental work and for a lack of methodological rigor.[3] This led to a movement to "rejuvenate" experimental CS, distinguishing between "building components" (prototyping) and "doing science" (experimenting).[3]

### 4.4.2 Benchmarking vs. Science

A common criticism in systems research is that "evaluation" often consists of ad-hoc benchmarks rather than hypothesis-driven inquiry. Evaluators should look for work that forms a clear hypothesis, constructs reproducible experiments to shed light on that hypothesis while controlling other variables, and analyzes the data to prove or disprove the claim.[40]

*Methodological rigor in systems requires:*

- **Workload Generation**: Creating realistic inputs that mimic real-world usage (e.g., using traces of actual web traffic).
- **Isolation of Variables**: Ensuring that observed differences are due to the system design and not background noise (e.g., OS jitter, network interference).
- **Reproducibility**: Ensuring that other researchers can run the same code and obtain the same results. This is a current crisis in the field, as many systems papers rely on proprietary code or hardware that is not available to others.[40]

# 5. Comparative Analysis: Tensions and Trade-offs

The coexistence of these three methodologies—Mathematical, Constructive, and Empirical—creates a vibrant but sometimes fractured discipline. Each has different values, validation criteria, and definitions of "success."

## 5.1 Comparing the Three Methodologies

The  table 4 summarizes the key differences between the three pillars.

## Table 4: Comparing the Three Methodologies

| Feature | Mathematical Modeling | Prototyping (Constructive) | Scientific Method (Empirical) |
|---|---|---|---|
| **Primary Goal** | Abstraction, Truth, & Provability | Utility, Innovation, & Feasibility | Explanation, Prediction, & Falsifiability |
| **Core Activity** | Formalizing, Proving, Abstracting | Designing, Building, Coding | Observing, Measuring, Testing |
| **Key Output** | Formal Models, Theorems, Proofs | Artifacts, Tools, Systems, Prototypes | Data, Plots, Accepted/Rejected Hypotheses |
| **Validation** | Deductive Proof (Correctness) | Feasibility (PoC), Utility, Novelty | Statistical Significance, Reproducibility |
| **Primary Risk** | "Gap" between model and reality | "Toy" solution (lacks scale/robustness) | Internal validity (uncontrolled variables) |
| **Typical Discipline** | Theoretical CS, Algorithms, Formal Methods | Systems, Software Engineering, DSR | HCI, Empirical SE, Performance Analysis |
| **Source of Truth** | Axioms and Logic | The Artifact itself (It works!) | Empirical Data (Observation) |

## 5.2 The Tension Between Theory and Practice

A significant tension exists between the "theoretical" (math-based) and "practical" (prototype-based) camps.[41]

- **Theory's Critique of Practice**: Theoreticians may view prototypes without formal proofs as "hacking"—unreliable, specific to one implementation, and lacking generalizable insight.[42] They argue that empirical testing can only show the presence of bugs, never their absence.[16]

- **Practice's Critique of Theory**: Practitioners often view formal models as "vacuous" because they rely on restrictive or false assumptions that ignore real-world constraints (e.g., memory caches, human behavior, unpredictable network latency).[43] A model that proves an algorithm is $O(n \log n)$ is useless if the constant factors make it too slow for real-time application.

This tension is often described as the "Gap" between formal specifications and running code.[18] A system might be verified correct against a model, but if the model simplifies the hardware too much, the system may still fail. Conversely, empirical results without a theoretical model may be non-generalizable—knowing *that* a system is faster is less useful than knowing *why*.[4]

## 5.3 Rigor vs. Relevance

In Constructive and Empirical research, there is a constant trade-off between rigor (control, precision) and relevance (realism, usefulness).[36]

- **High Rigor, Low Relevance**: A lab experiment where students use a simplified tool in a classroom allows for perfect control of variables but tells us little about how that tool would perform in a complex, messy industrial software development project.
- **Low Rigor, High Relevance**: A case study observing a team at a major tech company is highly relevant to industry but nearly impossible to control or replicate. The findings may be specific to that one company's culture.

The ABC Framework discussed in Section 4.2.2 explicitly acknowledges this, suggesting that researchers must choose their strategy based on which trade-off is acceptable for their specific research question.[36]

## 5.4 The "Valley of Death"

In Constructive Research, there is a phenomenon known as the "Valley of Death" for prototypes. This refers to the difficulty of transitioning a Research Prototype (which is often a "duct-taped" proof of concept) into an Industrial Prototype or product.[44] Academia incentivizes novelty and publication, not the robust engineering required to make a prototype production-ready. This gap often prevents promising research ideas from having a practical impact.

# 6. Synthesis and Integration: Toward a Unified Methodology

The most impactful computer science research often integrates these methodologies, creating a virtuous cycle of modeling, building, and evaluating. The interplay among these methodologies enriches the research landscape, offering diverse perspectives and tools for tackling challenges.[5]

## 6.1 Synergy: The Virtuous Cycle of Research

A robust research project often traverses all three pillars in a "virtuous cycle":

1. **From Model to Prototype**: A researcher uses mathematical modeling to design a new algorithm (e.g., a new consensus protocol) and verifies its safety properties formally.[45]
2. **From Prototype to Experiment**: The researcher implements this algorithm in a research prototype. This implementation reveals practical issues (e.g., unexpected network latency, memory overhead) that were ignored by the abstract model.[46]
3. **From Experiment to Theory**: Empirical benchmarking (scientific method) reveals unexpected behaviors or performance bottlenecks. The researcher analyzes the data to refine the mathematical model, making it more realistic and adding new constraints.[47]

This synergy is particularly evident in fields like Formal Verification of Systems, where researchers use formal methods to verify core components (like the seL4 microkernel) but rely on empirical testing for the un-modeled parts (such as hardware behavior or user-level applications).[48]

## 6.2 Mixed Methods Research

Increasingly, CS researchers employ Mixed Methods, combining qualitative and quantitative data to provide a more complete picture of a phenomenon.[50] This is distinct from simply using multiple algorithms; it involves mixing methodological paradigms.

- **Sequential Explanatory**: Running a quantitative experiment (e.g., a benchmark) and then conducting qualitative interviews with users or developers to understand *why* the results occurred.
- **Triangulation**: Using both logs (quantitative) and surveys (qualitative) to validate a finding from two different angles. If both the system logs and the user reports indicate a problem, the finding is much more robust.[52]

Mixed methods are particularly vital in HCI and Software Engineering, where "human factors" are as important as "system performance." For example, a new programming tool might be theoretically efficient (Model) and functionally complete (Prototype), but if developers find it confusing to use (Empirical/Qualitative), it will fail.[50]

## 6.3 Artifact Evaluation and Reproducibility

A major methodological shift in the last decade is the institutionalization of the "Constructive" methodology through Artifact Evaluation (AE) tracks at major conferences (e.g., POPL, OOPSLA, SIGMOD).[54] Historically, CS papers were judged solely on the text (the theory or the reported results). This encouraged "paper-ware"—systems that sounded good on paper but didn't actually exist or run.

Now, independent committees evaluate the artifact (the code, data, and scripts) to ensure it exists, runs, and supports the claims in the paper. This process:

- Validates the "Constructive" contribution: The artifact is recognized as a scholarly output in itself.

- Ensures "Scientific" reproducibility: It guarantees that the experiments described in the paper can be replicated.
- Bridges the "Theory-Practice Gap": It forces authors to make their implementation robust enough for others to use, moving it slightly out of the "research prototype" stage toward usability.

Papers that pass this process receive badges like "Artifact Functional" and "Artifact Reusable," signaling the quality of the engineering work underpinning the scientific claims.[54]

## 6.4 Future Directions: AI and Methodological Evolution

The rise of Artificial Intelligence (AI) and Machine Learning (ML) is introducing new methodological challenges. AI models are often "black boxes"—they are empirical objects that work (high predictive accuracy) but lack a theoretical model explaining *why* they work (low interpretability).[18]

This is shifting the field towards a more empirical direction, where "science" looks more like biology (observing the emergent behavior of a neural network) than mathematics (proving the correctness of an algorithm). Future research methodologies will likely focus on:

- **Explainable AI (XAI)**: Trying to derive mathematical models from empirical black boxes.
- **AI-assisted Formal Methods**: Using AI to help generate proofs, bridging the gap between intuition and formalism.[47]
- **Simulated Environments**: Using "Digital Twins" and high-fidelity simulations to prototype and test systems in silico before building physical artifacts.[56]

Methodological diversity is not a weakness of Computer Science but its defining strength. The field has evolved from "hacking" and pure mathematics into a sophisticated discipline that blends mathematical modeling to ensure correctness and abstraction, prototyping to demonstrate feasibility and innovation, and the scientific method to validate hypotheses and measure performance in the real world.

Successful computer science research often requires a traversal of all three pillars. A rigorous theoretical foundation provides the "why," a robust prototype provides the "how," and a careful empirical evaluation provides the "so what." As systems become more complex—incorporating AI, human behavior, and physical environments—the ability to triangulate findings using this tripartite methodological framework will become increasingly essential. The future of the field lies not in choosing between math, engineering, or science, but in mastering the synthesis of all three to solve the complex problems of the artificial world.

# Chapter bibliography


1. Abbasabadi, N., & Ashayeri, M. (2024). Artificial Intelligence in Performance-Driven Design: Theories, Methods, and Tools. In *Artificial Intelligence in Performance-Driven Design: Theories, Methods, and Tools*. https://doi.org/10.1002/9781394172092
2. Adhikari, M. S., Majumder, A., Sindhwani, M., & Sachdeva, S. (2025). AI and IoT integration for smart agriculture. In *Smart Electronic Devices: Artificial Intelligence, Machine Learning, and the Future*. https://doi.org/10.1201/9781003643173-3
3. Amblard, F., & Quattrociocchi, W. (2018). Simulating social complexity: Social Networks and Spatial Distribution. In *Simulating Complexity*.
4. Apiola, M., López-Pernas, S., & Saqr, M. (2023). Past, present and future of computing education research: A global perspective. In *Past, Present and Future of Computing Education Research: A Global Perspective*. https://doi.org/10.1007/978-3-031-25336-2
5. Baggio, R. (2018). Measuring tourism: Methods, indicators, and needs. In *The Future of Tourism: Innovation and Sustainability*. https://doi.org/10.1007/978-3-319-89941-1_13
6. Baggio, R. (2019). Measuring Tourism Methods, Indicators, and Needs: Innovation and Sustainability. In *The Future of Tourism*.
7. Brandes, U., & Erlebach, T. (2005). Network Analysis: Methodological Foundations. In *Lecture Notes in Computer Science* (Vol. 3418).
8. Breckling, B., Jopp, F., & Reuter, H. (2011). Historical background of ecological modelling and its importance for modern ecology. In *Modelling Complex Ecological Dynamics: An Introduction into Ecological Modelling for Students, Teachers & Scientists*. https://doi.org/10.1007/978-3-642-05029-9_3
9. Chen, S.-H., Terano, T., Yamamoto, R., & Tai, C.-C. (2014). Advances in Computational Social Science: The Fourth World Congress. In *Agent based social systems* (Issue 11).
10. Dahlberg, B., Wittink, M. N., & Gallo, J. J. (2015). Funding and Publishing Integrated Studies: Writing Effective Mixed Methods Manuscripts and Grant Proposals. In

*SAGE Handbook of Mixed Methods in Social & Behavioral Research*. https://doi.org/10.4135/9781506335193.n30

11. de Luca, L. (2023). A digital ecosystem for the interdisciplinary study of Notre-Dame de Paris | Un ecosistema digitale per lo studio interdisciplinare di Notre-Dame de Paris. In *Disegnare Idee Immagini* (Vol. 34, Issue 66).
12. de Roo Gert, Ward. S. Rauw. (2012). Springer Complexity program. In *Media* (Issue September 2007).
13. Edmonds, B., & Meyer, R. (2017). Simulating social complexity - A Handbook. In *Simulating Complexity*.
14. Frankhauser, P., Tannier, C., Vuidel, G., Houot, H., Kropf, K., Maretto, M., Strappa, G., Whitehand, J., Springer, & Tannier, C. (2018). About fractal models in urban geography and planning : refuting the aesthetics and the universal norm. In *Computers, Environment and Urban Systems* (Vol. 67, Issue 2017).
15. Friberg, E. E., & Saewert, K. J. (2023). Conceptual Foundations. In *Conceptual Foundations: The Bridge to Professional Nursing Practice*. https://doi.org/10.7551/mitpress/11171.003.0005
16. Gasperoni, F., Ieva, F., & Maria Paganoni, A. (2025). Exercise Book of Statistical Inference. In *UNITEXT - La Matematica per il 3 piu 2* (Vol. 173). https://doi.org/10.1007/978-3-031-86670-8
17. Geographic Citizen Science Design. (2021). In *Geographic Citizen Science Design*. https://doi.org/10.2307/j.ctv15d8174
18. Gratch, J., & Marsella, S. (2014). Social Emotions in Nature and Artifact. In *Social Emotions in Nature and Artifact*. https://doi.org/10.1093/acprof:oso/9780195387643.001.0001
19. Greco, C. (2024). Politics and practices of the ethnographies of biomedicine and STEM: Among white coats. In *Politics and Practices of the Ethnographies of Biomedicine and STEM: Among White Coats*. https://doi.org/10.1007/978-3-031-65797-9
20. Gross, T., & Sayama, H. (2009). Adaptive networks: Theory, models, and data. In *Springer Complexity Series*.

21. Haken, H. (2020). (Springer series in synergetics, Springer complexity) T.D. Frank - Nonlinear Fokker-Planck equations_ fundamentals and applications-Springer (2005).pdf. In *Media*.
22. Hardcastle, V. G. (2006). The theoretical and methodological foundations of cognitive neuroscience. In *Philosophy of Psychology and Cognitive Science*. https://doi.org/10.1016/B978-044451540-7/50027-X
23. Historical and Philosophical Dimensions of Logic, Methodology and Philosophy of Science. (1977). In *Historical and Philosophical Dimensions of Logic, Methodology and Philosophy of Science*. https://doi.org/10.1007/978-94-017-1780-9
24. Hoyte, S. (2021). Co-designing extreme citizen science projects in Cameroon: biodiversity conservation led by local values and indigenous knowledge. In *Geographic Citizen Science Design*. https://doi.org/10.2307/j.ctv15d8174.8
25. Jarke, J., Prietl, B., Egbert, S., Boeva, Y., Heuer, H., & Arnold, M. (2025a). Algorithmic Regimes: Methods, Interactions, and Politics. In *Algorithmic Regimes: Methods, Interactions, and Politics*. https://doi.org/10.4324/9781003690672
26. Jarke, J., Prietl, B., Egbert, S., Boeva, Y., Heuer, H., & Arnold, M. (2025b). Algorithmic Regimes: Methods, Interactions, and Politics. In *Algorithmic Regimes: Methods, Interactions, and Politics*. https://doi.org/10.4324/9781003690672
27. Koumpis, A. (2012). Management Information Systems for Enterprise Applications: Business Issues, Research and Solutions. In *Management Information Systems for Enterprise Applications: Business Issues, Research and Solutions*. https://doi.org/10.4018/978-1-4666-0164-2
28. Kroemer, K. H. E. (1989). A Survey of Ergonomic Models of Anthropometry, Human Biomechanics, and Operator-Equipment Interfaces. In *Applications of Human Performance Models to System Design*. https://doi.org/10.1007/978-1-4757-9244-7_23
29. Kulas, J., Fetzer, J. H., & Rankin, T. L. (2011). Philosophy, Language, and Artificial Intelligence: Resources for Processing Natural Language. In *Studies in Cognitive Systems*.
30. Lamas, D., & Tomberg, V. (2024). HCI Design Perspectives, Trends and Approaches.

In *Foundations and Fundamentals in Human-Computer Interaction*. https://doi.org/10.1201/9781003495109-11

31. Marquis, P., Papini, O., & Prade, H. (2020). A Guided Tour of Artificial Intelligence Research: Volume III: Interfaces and Applications of Artificial Intelligence. In *A Guided Tour of Artificial Intelligence Research: Volume III: Interfaces and Applications of Artificial Intelligence* (Vol. 3). https://doi.org/10.1007/978-3-030-06170-8
32. Mathematical Methods in Signal Processing and Digital Image Analysis. (2008). In *Mathematical Methods in Signal Processing and Digital Image Analysis*. https://doi.org/10.1007/978-3-540-75632-3
33. Moss, L. S., & Johnson, D. E. (1995). Evolving Algebras and Mathematical Models of Language. In *Applied Logic: How, What and Why*. https://doi.org/10.1007/978-94-015-8533-0_7
34. Niglas, K. (2015). The Multidimensional Model of Research Methodology: An. In *SAGE Handbook of Mixed Methods in Social & Behavioral Research*.
35. *Nonlinear resonance*. (2015). https://doi.org/10.1090/gsm/167/08
36. Nyerges, T. L., Couclelis, H., & McMaster, R. (2011). The SAGE handbook of GIS and society. In *The SAGE Handbook of GIS and Society*. https://doi.org/10.4135/9781446201046
37. Ocaña, A. C., Mengual-Andres, S., & Segarra, J. R. (2024). Educational innovation to address complex societal challenges. In *Educational Innovation to Address Complex Societal Challenges*. https://doi.org/10.4018/9798369330739
38. Oshima, J., van Aalst, J., Mu, J., & Chan, C. K. K. (2023). Development of the learning sciences: Theories, pedagogies, and technologies. In *International Handbook on Education Development in the Asia-Pacific*. https://doi.org/10.1007/978-981-19-6887-7_49
39. Partos, R. (2019). Time to legislate. In *The Making of the Conservative Party's Immigration Policy*. https://doi.org/10.4324/9781351010658-3
40. Petrucco, C., & Agostini, D. (2019). Augmented Reality Learning: Pedagogical Aspects and Technologies for a Future Methodological Framework. In *Virtual*

*Worlds: The Virtual Reality and Augmented Reality Intersections*. https://doi.org/10.1163/9781848883840_007

41. Portugali, J. (2012). Complexity Theories of Cities: Implications to Urban Planning. In *Complexity Theories of Cities Have Come of Age*. https://doi.org/10.1007/978-3-642-24544-2_13
42. Primiero, G. (2019). On the Foundations of Computing. In *On the Foundations of Computing*. https://doi.org/10.1093/oso/9780198835646.001.0001
43. Pumain, D. (2012). Urban Systems Dynamics, Urban Growth and Scaling Laws: The Question of Ergodicity. In *Complexity Theories of Cities Have Come of Age*. https://doi.org/10.1007/978-3-642-24544-2_6
44. Pyka, A., & Scharnhorst, A. (2009). Innovation Networks; New Approaches in Modelling and Analyzing. In *Springer*.
45. Ringquist, E. J., & Anderson, M. R. (2013). Introduction. Meta-analysis for Public Management and Policy. In *Meta-Analysis for Public Management and Policy.*
46. Rossy, Q., & Morselli, C. (2017). The contribution of forensic science to the analysis of crime networks. In *The Routledge International Handbook of Forensic Intelligence and Criminology*. https://doi.org/10.4324/9781315541945
47. Stangl, F. J., & Riedl, R. (2025). *Hormone Measurements in the Research Field of Interruption Science: Review of Applied Methods*. https://doi.org/10.1007/978-3-031-71385-9_8
48. Tashakkori, A., & Teddlie, C. (2015). Epilogue: Current Developments and Emerging Trends in Integrated Research Methodology. In *SAGE Handbook of Mixed Methods in Social & Behavioral Research*. https://doi.org/10.4135/9781506335193.n31
49. Wang, W. (2024). Principles of Machine Learning: The Three Perspectives. In *Principles of Machine Learning: the Three Perspectives*. https://doi.org/10.1007/978-981-97-5333-8
50. Zamlynskyi, V., Kalinichenko, S., Kniazkova, V., Skrypnyk, N., & Avriata, A. (2025). Healthcare management using artificial intelligence. In *Revolutionizing Medical Systems Using Artificial Intelligence: A Breakthrough in Healthcare*.

https://doi.org/10.1016/B978-0-443-32862-6.00001-8

51. Alwi, Z. (2016). studi ilmu hadist. In *Revista Brasileira de Linguística Aplicada* (Vol. 5, Issue 1).
52. Bransford, J. D., Barron, B., Pea, R. D., Meltzoff, A., Kuhl, P., Bell, P., Stevens, R., Schwartz, D. L., Vye, N., Reeves, B., Roschelle, J., & Sabelli, N. H. (2006). Foundations and opportunities for an interdisciplinary science of learning. In *The Cambridge handbook of the learning sciences*.
53. C. B., & N. J. S. (2019). Computer Simulation Validation Fundamental Concepts, Methodological Frameworks, and Philosophical Perspectives. In *Simulation Foundations, Methods and Applications*.
54. Mutanov, G. (2015). Mathematical Methods and Models in Economic Planning, Management and Budgeting. In *Mathematical Methods and Models in Economic Planning, Management and Budgeting*. https://doi.org/10.1007/978-3-662-45142-7
55. Rubik, Programa Construye2025, Ertuğ KIYASOĞLU, UNEP, ONU Medio Ambiente, Minson, A. J., Latinoamericano, F., European Commission, E., Normativo, T., Nacional, C., Priess, J. A., Hauck, J., Haines-Young, R., Alkemade, R., Mandryk, M., Veerkamp, C., Gyorgyi, B., Dunford, R., Berry, P., … Europeiska Kommissionen. (2019). Hoja de Ruta RCD: Consulta Pública - Gestión Sustentable de los Recursos y Residuos, RCD, para una Economía Circular en Construcción. In *European Comission* (Vol. 53, Issue 1).
56. Davis, G. B. (1985). Management Information System: Conceptual Foundation, Structure, and Development (Second). In *Mc. Graw-Hill, Inc*.

# Chapter III.

# Mathematical Modeling and the Educational Research Process: Theoretical Foundations, Methodological Architectures, and Empirical Trajectories

## 1. Epistemological Foundations of the Models and Modeling Perspective

The trajectory of mathematics education research over the past three decades has been characterized by a decisive shift away from the "acquisition metaphor" of learning—which views knowledge as a commodity to be transmitted and stored—toward a perspective that emphasizes the development of conceptual systems. This paradigm shift is most rigorously articulated in the Models and Modeling Perspective (MMP). The MMP fundamentally reconfigures the ontology of mathematical learning, positing that the primary goal of instruction is not merely the mastery of isolated procedural skills or the memorization of algorithms, but the cultivation of powerful, shareable, and reusable conceptual models that enable learners to describe, explain, and predict the behavior of complex real-world systems.[1]

This perspective emerged as a response to the limitations of traditional constructivism. While constructivism successfully argued that learners actively build knowledge, it often lacked a specific description of *what* is being constructed and how those constructions evolve in the context of complex problem-solving. The MMP fills this void by identifying "models" as the fundamental unit of cognition.[3] In this view, models are not static mental pictures but dynamic conceptual tools consisting of elements, relationships, operations, and rules that learners project onto their experiences to make sense of them.[1]

## 1.1 The Nature of Models in Educational Contexts

Within the MMP framework, a model is understood as a system of thinking that is fundamentally social and iterative. Unlike the traditional definition of a mathematical model—which might be limited to a set of differential equations or a statistical regression—the educational definition is broader. It encompasses the internal conceptual systems (mental models) and the external representations (graphs, diagrams, equations, metaphors) that learners use to externalize their thinking.[1]

The distinction between internal and external representation is critical for the research process. Internal conceptual systems are inaccessible to direct observation. However, when students are engaged in Model-Eliciting Activities (MEAs), they are compelled to externalize these internal systems into visible artifacts. This process of externalization serves a dual purpose: it stabilizes the student's own thinking, allowing for self-reflection and revision, and it provides researchers with a tangible "audit trail" of cognition.[4] Thus, the model becomes the interface between the private world of the learner's mind and the public world of social negotiation and educational assessment.

The iterative nature of these models is another defining characteristic. Models are rarely formed in a finished state. Instead, they evolve through a series of "modeling cycles." Learners begin with primitive, unstable, and often barren interpretations of a problem situation. Through interaction with the problem's constraints and with peer feedback, these initial models are tested, revised, differentiated, and integrated into more robust and stable systems.[5] This evolution mimics the scientific process itself, where theories are proposed, tested against empirical data, and refined.

## 1.2 Holistic vs. Atomistic Approaches to Learning

The MMP challenges the "atomistic" curriculum design prevalent in many educational systems, which breaks mathematics down into discrete, decontextualized skills (atoms) to be mastered sequentially. The assumption behind atomistic instruction is that once students have collected enough "atoms" of knowledge, they will spontaneously assemble them into complex understanding. Empirical evidence suggests this rarely happens; students often fail to transfer these isolated skills to novel contexts.[2]

In contrast, the MMP advocates for a holistic approach where the "whole" (the model) gives meaning to the "parts" (the specific mathematical procedures). A student does not learn about ratios in isolation and then apply them; rather, the need to quantify the steepness of a ramp or the density of a population *creates the need* for a ratio model. The model provides the structural context in which specific mathematical tools (arithmetic operations, algebraic notation) become meaningful and necessary.[2] This aligns with findings that students demonstrate significantly higher engagement and retention when mathematics is presented as a tool for solving authentic, client-driven problems rather than as a set of abstract rules.[8]

## 1.3 Mathematical Modeling as a Research Methodology

Perhaps the most significant contribution of the MMP is its integration of instructional design and research methodology. In traditional educational research, the "treatment" (instruction) and the "measurement" (testing) are separate events. The researcher teaches a concept, then administers a post-test to see if it stuck. This approach often fails to capture the *process* of learning, offering only a snapshot of the *product*.

The MMP dissolves this dichotomy. The Model-Eliciting Activity (MEA) acts simultaneously as the instructional vehicle and the research instrument. Because MEAs require students to document their decision-making processes and justify their solutions to a client, the "data" is generated in real-time as the learning occurs.[4] This "thought-revealing" quality allows researchers to trace the genesis of mathematical ideas—from their messy, informal origins to their formal, mathematical expressions—without the interference of artificial testing protocols.[10]

The following table summarizes the fundamental distinctions between traditional problem-solving research and the Models and Modeling Perspective (see Table 5).

### Table 5: Traditional Problem Solving Research Vs. Models and Modeling Perspective (MMP)

| Dimension | Traditional Problem Solving Research | Models and Modeling Perspective (MMP) |
|---|---|---|
| **Unit of Analysis** | Isolated skills or procedural fluency. | Conceptual systems (models) and their evolution. |
| **View of Learning** | Acquisition of facts and procedures (Linear). | Iterative development of models (Recursive/Zig-Zag). |
| **Problem Context** | Well-defined, "puzzle-like" problems with single answers. | Ill-structured, complex, real-world situations with multiple solutions. |
| **Role of Social Interaction** | Often viewed as "cheating" or noise; focus on individual cognition. | Essential; models are co-constructed and negotiated in groups. |
| **Research Data** | Pre-test and Post-test scores (Outcome-based). | Transcripts, student drafts, and final model documentation (Process-based). |
| **Goal of Instruction** | Mastery of curriculum standards. | Development of shareable, reusable, and transferable conceptual tools. |
| **Validation Authority** | The teacher or the answer key. | The problem constraints and the "client's" needs (Self-assessment). |

## 2. The Architecture of Model-Eliciting Activities (MEAs)

The operational heart of the MMP is the Model-Eliciting Activity (MEA). These tasks are meticulously engineered to simulate real-world professional contexts, such as engineering, urban planning, or business consulting. Unlike standard "word problems," which typically require the application of a previously taught procedure, MEAs are designed to be "model-eliciting"—that is, they require the learner to invent, extend, or refine a mathematical system to solve the problem.[9]

The design of these activities is governed by six fundamental principles, first articulated by Lesh, Hoover, Kelly, and Post (2000). These principles serve as a quality control mechanism for instructional design and as a validity check for research data. If a task violates these principles, it is unlikely to reveal significant insights into student thinking.[12]

## 2.1 The Reality Principle

The Reality Principle dictates that the problem context must be meaningful and accessible to the students based on their existing life experiences. This does not mean the problem must be "real" in the sense of happening right now, but it must be *realistic*—students must be able to imagine the situation and care about the outcome.[12]

Crucially, the Reality Principle ensures that students can use their "real-world" knowledge to validate their mathematical results. In traditional textbook problems (often critiqued as "pseudo-contexts"), the reality is stripped away; students learn to ignore common sense (e.g., assuming a bus can travel at constant velocity forever). in an MEA, reality is the first line of defense against error. If a model predicts that a "Giant" is 40 feet tall based on a footprint, the Reality Principle compels the student to pause and reject that conclusion based on their knowledge of human biology.[12] This grounding in reality lowers the barrier to entry (the "low floor") while allowing for sophisticated mathematical analysis (the "high ceiling").[2]

## 2.2 The Model Construction Principle

This principle mandates that the task must require the creation of a *system*, not just the calculation of a value. The question asked of the student should not be "What is the answer?" but "How should this be decided?" or "What rule works best?".[12] For example, in the "Summer Jobs" MEA, students are not asked to calculate the sum of wages. They are asked to develop a method for hiring the "best" employees based on conflicting data sets involving hours worked, money earned, and consistency.[15] This shift forces students to construct a model that defines "best"—is it total accumulation? Is it efficiency (earnings per hour)? Is it reliability? The resulting product is a decision-making algorithm, which is a mathematical model in its purest form.[2]

## 2.3 The Self-Assessment Principle

Perhaps the most critical principle for fostering autonomy, the Self-Assessment Principle requires that the problem statement contains the criteria for success. Students should be able to judge for themselves whether their solution is good enough without asking the teacher, "Is this right?".[4]

In practice, this is often achieved through "test data" provided by a fictional client. If the students' model works for Employee A but fails for Employee B, the data itself provides the feedback. This feedback loop drives the iterative nature of the modeling cycle. Students test their model, see it fail against the data, and revise it. This creates a "need for revision" that comes from the task, not the instructor, shifting the locus of authority from the teacher to the mathematics itself.[12]

## 2.4 The Model Documentation Principle

The Model Documentation Principle ensures that the students' thinking is made visible. The task must explicitly require students to describe their process, usually in the form of a memo, letter, or "user guide" to the client.[4]

This principle is the bridge between instruction and research. Without documentation, a researcher might see a correct answer but have no idea how it was derived. By requiring a written explanation of the "toolkit" or "procedure," the MEA forces students to externalize their internal cognitive state. This documentation serves as the primary data source for researchers, allowing for the analysis of the *process* of model development rather than just the final *product*.[4]

## 2.5 The Generalizability (Shareability) Principle

Models are powerful because they are reusable. The Generalizability Principle states that the solution created by the students should not just work for the specific instance in the problem (e.g., this specific footprint), but should be shareable with others and usable in similar situations (e.g., any footprint found at a crime scene).[4]

This pushes students toward mathematical abstraction. A solution that says, "It's 10 feet tall because I measured it with my finger" is not shareable. A solution that says, "Measure the foot length, multiply by 6.5, and add 2 inches" is a generalizable algorithm. This principle forces students to move from ad-hoc reasoning to formal mathematical syntax, facilitating the transfer of learning to new contexts.[18]

## 2.6 The Effective Prototype Principle

The final principle ensures that the resulting model is simple yet powerful—a "prototype" or metaphor that students can carry forward to interpret other situations.[12] If the model is overly complex or messy, it is less likely to be retained as a cognitive tool. An effective prototype serves as a mental hook. For instance, once students develop a "rate of change" model to solve a problem about filling a water tank, that model becomes a prototype they can apply to problems about population growth or velocity. The MEA acts as the genesis event for this conceptual prototype.[2]

# 3. The Modeling Cycle: Cognitive Dynamics and

# Student Trajectories

Learning in the MMP is conceptualized as a "Modeling Cycle." This cycle is rarely linear; it is a "zig-zag" path of development where students move through phases of expressing, testing, and revising their thinking.[6] Researchers have mapped these cycles extensively, identifying how groups negotiate the conflict between their initial intuitive ideas and the formal demands of the problem.

## 3.1 Phases of the Modeling Cycle

1. **Interpretation and Expressing:** Students first attempt to understand the problem situation. They filter the messy data, identifying which variables matter. This often results in a "barren" or overly simple initial model.
2. **Mathematization and Testing:** Students attempt to map their initial ideas onto mathematical structures (e.g., deciding to add numbers or find an average). They then "run" this model against the data provided in the problem.
3. **Revision and Refinement:** The testing phase usually reveals flaws. The model might give a ridiculous answer (violating the Reality Principle) or fail to account for a specific data point. This failure triggers a revision phase, where students differentiate their concepts (e.g., realizing that "big" can mean "tall" or "heavy") or integrate new variables.[2]

## 3.2 Case Study Analysis: The Bigfoot MEA

The "Bigfoot" MEA is a classic example used in MMP research to illustrate the evolution from qualitative to multiplicative reasoning. In this activity, students are provided with a photograph of a large footprint and asked to develop a "toolkit" for police to estimate the height of the person who made it.[14]

*Phase 1: Qualitative and Intuitive Reasoning*

Research transcripts reveal that student groups often begin with purely descriptive, non-mathematical observations.

- *Student Quote:* "Wow! This guy is huge... You know any girls that big?".[14]
- Student Quote: "Those're Nike's... The tread's just like mine."
  At this stage, the students are engaging with the context but have not yet formulated a mathematical model. They are using their personal experiences (Reality Principle) to orient themselves.

*Phase 2: Additive Reasoning (The First Primitive Model)*

As they move to quantification, students frequently default to additive strategies, which are cognitively less demanding than proportional ones.

- *Observed Strategy:* A student places his own foot next to the footprint. He uses his fingers to mark the gap—let's say it's 3 inches. He then reasons, "His foot is 3 inches longer than mine, so he must be 3 inches taller than me".[14]
- *Testing and Failure:* When the group tests this model (Self-Assessment), they realize that a person only 3 inches taller than a middle schooler is not a "giant." The model produces a result that violates the "Bigfoot" premise. This cognitive dissonance—the failure of the additive model—is the catalyst for the next leap in learning.

*Phase 3: Proportional Reasoning (The Paradigm Shift)*

Forced to abandon the additive model, students search for a relationship that preserves the "bigness."

- *Emerging Insight:* "It's not about how much longer, it's about how many *times* longer."
- *Mathematization:* Students begin to calculate ratios. "My foot is size 8 and I am 5 feet tall. This foot is size 16. That's double. So he must be double my height."
- *Refinement:* This proportional model (Height = k \times FootLength) is robust. Students then refine it by gathering data from the whole class to find an average ratio (k \approx 6.6), satisfying the Generalizability Principle.[18]

This trajectory demonstrates that proportional reasoning is not just "applied" but "invented" in response to the limitations of additive thinking.

## 3.3 Case Study Analysis: The Summer Jobs MEA

The "Summer Jobs" MEA exposes the conflict between different conceptual definitions of value and the difficulty of quantifying them. The problem asks students to select the best employees based on data tables of earnings and hours worked.[15]

*The Conceptual Conflict: Accumulation vs. Efficiency*

Research on this MEA highlights a divergence in student hypotheses:

- **Veli's Hypothesis (Accumulation Model):** "The time is not important. I think the ones earning the highest amount of money should be the best." This model posits that *Total Value* = *Total Earnings*. It ignores the cost of time.
- **Sıla's Hypothesis (Efficiency Model):** "We will also find the one who brings the highest amount of money in the shortest time." This model introduces a rate: *Value* = *Earnings / Time*.

*The Modeling Paradox: Conceptual Agreement vs. Mathematical Execution*

The group conceptually agreed that Sıla's efficiency model was superior. However, when they moved to the "mathematization" phase to create a scoring system, a critical error occurred.

- *The Scoring System:* The students decided to award points: 3 points for the highest earnings, and *also* 3 points for the *longest* working hours.
- *The Contradiction:* By awarding points for long hours, they mathematically penalized efficiency—contradicting their own conceptual agreement.
- *The Outcome:* A worker named Ahmet received the maximum score (3 points for earnings + 2 points for other factors) because he worked long hours, even though he was inefficient.
- *Student Reflection:* Veli remarked, "This scoring system has good mathematics. I am glad I found a scoring system".[15]

This finding is profound for educational researchers. It illustrates "the seduction of quantification." The students were so relieved to have a system that produced numbers ("good mathematics") that they failed to notice the numbers contradicted their logic. This disconnect between the *conceptual model* and the *computational model* is a fertile ground for teacher intervention and learning.[15]

# 4. Methodology: The Multi-Tiered Teaching Experiment

To capture the complexity of these modeling cycles, MMP researchers utilize a specialized methodological design known as the "Multi-Tiered Teaching Experiment." This design acknowledges that educational settings are complex systems where learning occurs simultaneously at multiple levels: the student level, the teacher level, and the researcher level.[4]

## 4.1 Tier 1: The Student Tier

At the first tier, the focus is on the students interacting with the real-world context provided by the MEA.

- **Agent:** Students.
- **Task:** Model-Eliciting Activity (e.g., Bigfoot).
- **Goal:** Construct a model of the physical reality.
- **Data Generated:** Audio/video transcripts of group discussions, draft notes, final client reports.
- **Research Focus:** Tracking the evolution of mathematical concepts (e.g., from additive to proportional).[23]

## 4.2 Tier 2: The Teacher Tier

The second tier focuses on the teacher. Just as students must model the Bigfoot problem, teachers must model the students' thinking.

- **Agent:** Teachers.

- **Task:** Model-Eliciting Activities for Teachers (MEA-Ts). For example, a teacher might be given a set of student responses to the Summer Jobs problem and asked to design an assessment rubric.
- **Goal:** Construct a model of student cognition and pedagogy.
- **Data Generated:** Teacher interview transcripts, lesson plans, teacher reflections on student work.
- **Research Focus:** How do teachers interpret student errors? Do they see them as mistakes to be corrected or as windows into developing logic?.[4]

## 4.3 Tier 3: The Researcher Tier

The third tier involves the researchers themselves.

- **Agent:** Researchers.
- **Task:** Analyzing the interactions between Tier 1 and Tier 2.
- **Goal:** Construct a model of the entire educational ecosystem (theory building).
- **Data Generated:** Academic papers, theoretical frameworks, coding schemes.
- **Research Focus:** Developing generalizable theories about learning and instruction.[22]

This recursive design ensures that the research is not extractive but collaborative. Researchers are not just observing "subjects"; they are participants in a system where their own theories are constantly being tested and revised against the reality of the classroom.[7]

## 4.4 Data Analysis: Protocol Analysis and Triangulation

Analyzing the massive amount of qualitative data generated by these experiments requires rigorous protocols.

- **Protocol Analysis:** Researchers parse transcripts of student dialogue into "episodes" of reasoning. They look for specific "markers" of cognitive shifts, such as changes in language (e.g., from "it looks like" to "the data shows").[24]
- **Coding Schemes:** Detailed coding schemes are developed to categorize student

statements. For example, in the Summer Jobs problem, codes might distinguish between "absolute comparisons" (Employee A made 500) and "relative comparisons" (Employee A made 10/hour).[25]

- **Triangulation:** To ensure validity, findings are triangulated across data sources. A researcher's interpretation of a student's confusion is cross-referenced with the teacher's field notes and the student's own written reflection. If all three align, the finding is considered robust.[12]
- **Inter-Rater Reliability:** When multiple researchers code transcripts, they must achieve a high level of agreement (typically >70% or >0.8 Cohen's Kappa). Disagreements are resolved through discussion, which further refines the coding definitions.[12]

# 5. Quantitative Assessment and Engineering Education Outcomes

While the MMP is rooted in qualitative inquiry, it bridges the gap to quantitative assessment, particularly in the context of engineering education and ABET (Accreditation Board for Engineering and Technology) accreditation. MEAs are increasingly used in undergraduate engineering courses to measure "professional skills" that are difficult to capture with traditional exams.[8]

## 5.1 The Quality Assurance Guide (QAG)

To quantify performance on MEAs, researchers developed the Quality Assurance Guide (QAG). This rubric assesses student products not on "right/wrong" binary scales, but on the *quality* and *utility* of the model (see Table 6).[30]

### Table 6: Dimensions of the Quality Assurance Guide (QAG)

| Dimension | Description | Score Range |
| --- | --- | --- |

| | | |
|---|---|---|
| **Usefulness** | Does the solution meet the client's needs? Is it practical? | 1 (Not useful) - 5 (Very useful) |
| **Justification** | Are the decisions supported by data and mathematical reasoning? | 1 (No justification) - 5 (Fully justified) |
| **Shareability** | Can the model be understood and used by others without the authors present? | 1 (Not shareable) - 5 (User-friendly) |
| **Reusability** | Can the model be applied to similar problems with different data? | 1 (One-time use) - 5 (Generalizable) |

Studies utilizing the QAG have demonstrated that students who engage in MEAs show significant improvements in problem-solving scores over the course of a semester compared to control groups.[30]

## 5.2 Measuring Incremental Validity and ABET Outcomes

Engineering programs often struggle to assess "soft skills" like teamwork, ethical reasoning, and communication (ABET outcomes). Research indicates that MEAs provide a valid instrument for these measures.

- **Incremental Validity:** A study by Kim & Moore (2019) found that MEA scores added significant predictive capacity to models of student success, explaining variance that GPA and standardized test scores could not. This suggests that MEAs measure a distinct construct—"modeling competency" or "engineering design thinking"—that is independent of rote academic ability.[33]
- **Pre/Post Concept Inventories:** When MEAs are paired with Concept Inventories (tests of fundamental conceptual understanding), researchers observe that MEAs help students repair misconceptions. The iterative testing phase of the MEA forces students

to confront their misunderstandings in a way that lectures do not.[8]

# 6. The Teacher's Role and Professional Development

The implementation of MMP places significant demands on teachers, requiring a shift from "implementer" to "investigator." The "Multi-Tiered" design explicitly includes teachers as learners, acknowledging that they too must develop new models of pedagogy.[28]

## 6.1 The Challenge of "Letting Go"

Traditional teaching often involves "smoothing the path" for students—removing obstacles to ensure they reach the correct answer efficiently. In the MMP, obstacles are the engine of learning. Teachers must learn to "let go" and allow students to struggle with the ambiguity of the MEA.[2]

- **Didactic Tension:** Teachers often feel a strong urge to intervene when they see students heading down a "wrong" path (like Veli's accumulation model). However, MMP research shows that premature intervention robs students of the opportunity to self-correct via the Self-Assessment Principle.
- **Pedagogical Content Knowledge:** Facilitating an MEA requires a deep understanding of the multiple ways a problem *can* be solved. A teacher must be prepared to evaluate a graphical solution, an algebraic solution, and a statistical solution simultaneously.[28]

## 6.2 Model-Eliciting Activities for Teachers (MEA-Ts)

To support this shift, professional development programs utilize MEA-Ts. These are simulations where teachers are the "students."

- **Example Task:** Teachers are given a packet of "Student Work" (responses to the Bigfoot problem ranging from poor to excellent) and a "Client Request" from the school principal: "Design a grading rubric to assess these students' mathematical thinking."
- **Outcome:** By working together to build the rubric (a model), teachers must externalize and negotiate their beliefs about what constitutes "good" mathematics. Do they value

the correct number? Or the reasoning process? This activity mirrors the student's modeling cycle, allowing teachers to experience the same intellectual struggle.[1]

### 6.3 Shifting Beliefs and Dispositions

Research confirms that engagement with MEAs changes teacher dispositions. Teachers who use MEAs tend to move away from "deficit models" (focusing on what students lack) toward "competency models" (recognizing the diverse resources students bring). They begin to see student errors not as failures of memory, but as rational attempts to model complexity based on limited data.[5]

## 7. Technology, STEM, and Future Directions

The future of MMP research is increasingly intertwined with technology and interdisciplinary STEM education.

### 7.1 Technology as a Modeling Tool

Digital tools like GeoGebra, dynamic spreadsheets, and simulation software act as "amplifiers" for student modeling.

- **Dynamic Modeling:** In calculus and algebra, tools like GeoGebra allow students to manipulate parameters and instantly see the effect on the model (e.g., changing the slope of a line). This tightens the feedback loop of the modeling cycle, allowing for more rapid iterations of testing and revision.[19]
- **Data Analysis:** For MEAs involving large datasets (like the Summer Jobs problem), spreadsheets enable students to handle complexity that would be impossible by hand. The ability to sort, filter, and graph data allows students to move beyond arithmetic and engage in statistical reasoning.[32]

### 7.2 Interdisciplinary STEM Connections

MEAs are naturally interdisciplinary. The "Bigfoot" problem connects biology (proportions) with math; the "Summer Jobs" problem connects economics (efficiency vs. cost) with data analysis.

- **STEM Integration:** Research shows that MEAs are effective vehicles for STEM integration because they require the simultaneous application of knowledge from multiple domains. A study by Armutcu and Bal (2017) demonstrated that MEAs improved students' ability to synthesize science and math concepts to solve engineering challenges.[11]
- **Science Literacy:** MEAs that tackle controversial topics (e.g., environmental data, pseudoscience like Bigfoot) provide a platform for teaching scientific literacy and critical thinking. Students learn to question the source of data, the validity of assumptions, and the limits of models—skills essential for navigating the modern world.[37]

## 7.3 Policy and Assessment Implications

The greatest barrier to the widespread adoption of MMP is the misalignment with standardized testing. Most high-stakes tests value speed and procedural accuracy, whereas MEAs value depth, iteration, and revision.

- **The "Assessment Gap":** Research indicates that while MEAs improve deep conceptual understanding, these gains do not always translate to higher scores on multiple-choice standardized tests, which measure different constructs. This creates a policy dilemma: schools are incentivized to teach to the test, potentially at the expense of the modeling competencies required for the 21st-century workforce.[38]
- **Future Research:** A critical area for future MMP research is the development of scalable, standardized assessments that can measure modeling competency reliably, providing a viable alternative to traditional testing.[34]

The Models and Modeling Perspective represents a maturing field of educational inquiry that offers a robust alternative to traditional instructional and research paradigms. By centering the "model" as the fundamental unit of analysis, the MMP provides a theoretical framework that accounts for the complex, non-linear, and social nature of human learning.

The methodological innovations of the MMP—specifically the Model-Eliciting Activity (MEA) and the Multi-Tiered Teaching Experiment—have dissolved the artificial boundary between instruction and assessment. MEAs serve as powerful lenses, revealing the microscopic evolution of student thinking from intuitive guesses to sophisticated mathematical systems. The case studies of "Bigfoot" and "Summer Jobs" serve as empirical validation of this framework, demonstrating that even young students are capable of profound mathematical invention when the task structure supports their autonomy.

However, the path forward is not without challenges. The rigorous demands of qualitative analysis, the need for intense teacher professional development, and the friction with current assessment policies present significant hurdles. Yet, as the demand for a workforce capable of navigating complex systems grows, the relevance of the Models and Modeling Perspective only increases. It offers not just a way to teach mathematics, but a way to research the very nature of how human beings organize their experience of the world.

# Chapter bibliography


1. Analysis of international experience in implementing Artificial Intelligence in the educational process. (2025). *Scientia et Societus*. https://doi.org/10.69587/ss/1.2025.76
2. Apaza, I. M. A., Choque, H. W. Z., & Laqui, A. J. C. (2025). Transforming university education: a systematic review of mathematical modeling in learning. *International Journal of Evaluation and Research in Education*, *14*(3). https://doi.org/10.11591/ijere.v14i3.32523
3. Baimakhanova, A. B., Kulton, Y., & Rakhimzanova, L. B. (2025). Training of future doctors in computer modeling. *R&E-SOURCE*. https://doi.org/10.53349/re-source.2025.is1.a1373
4. Barbosa, M. I. G., Senna, M. L. G. S. de, Dias, R. C., Cavalcante, R. P., & Castilho, W. S. (2025). MODELAGEM MATEMÁTICA E FORMAÇÃO OMNILATERAL – ESTUDO DE CASO SOBRE O PROCESSO DE ENSINO-APRENDIZAGEM DE MATEMÁTICA NO ENSINO MÉDIO INTEGRADO. *Revista Ibero-Americana de Humanidades, Ciências e Educação*, *11*(5). https://doi.org/10.51891/rease.v11i5.19118
5. Bilan, N., Volkova, N., Fedoreiko, I., Horbatiuk, R., Bureha, N., Zahorodnii, R., & Zamora, Y. (2025). Methodology of foreign language training of master's students in the field of energy using innovative technologies. *Edelweiss Applied Science and Technology*, *9*(3). https://doi.org/10.55214/25768484.v9i3.5154
6. Biryukov, V. (2025). The importance of hierarchy and classification of sciences in the educational process. *International Science Journal of Education & Linguistics*, *4*(3). https://doi.org/10.46299/j.isjel.20250403.02
7. Brega, O. N., & Kruglyakova, G. v. (2025). Digital competence development while video podcast creation in ESP. *Perspektivy Nauki i Obrazovania*, *74*(2). https://doi.org/10.32744/pse.2025.2.14
8. Brovko, K., Vynokurova, N., & Velykohorskyi, O. (2025). Computer simulation of the development of the Chornobyl nuclear power plant accident using an interactive simulator. *Bulletin of the National Technical University "KhPI". Series: Energy: Reliability*

*and Energy Efficiency*, *1(10)*. https://doi.org/10.20998/eree.2025.1(10).329980

9. Bystrushkina, N., Vasilyeva, T., & Holina, S. (2025). The style of self-regulation of behavior among students of a medical university engaged in sports. *Scientific Notes of P. F. Lesgaft University*. https://doi.org/10.5930/1994-4683-2025-254-260
10. Chigambayeva, D., Goryakin, M., & Batova, P. (2025). EDUCATIONAL FLOWS DISTRIBUTION SYSTEM OF UNIVERSITY ACADEMIC GROUPS. *Scientific Journal of Astana IT University*. https://doi.org/10.37943/22vhyr8574
11. Chigambayeva, D., & Soltan, G. (2025). CONSTRACTION OF DISTRIBUTION MODELS OF THE UNIVERSITY EDUCATIONAL WORK VOLUME. *Scientific Journal of Astana IT University*. https://doi.org/10.37943/21hivr6985
12. Chushchak, I., & Andrunyk, V. (2025). Modeling an information technology system to support the educational process for individuals with special needs. *Vìsnik Nacìonal'nogo Unìversitetu "L'vìvs'ka Polìtehnìka". Serìâ Ìnformacìjnì Sistemi Ta Merežì*, *17*. https://doi.org/10.23939/sisn2025.17.330
13. Despoina G. Mavridou. (2025). The necessity of modeling at school and the teacher's role. *International Journal of Science and Research Archive*, *16*(2). https://doi.org/10.30574/ijsra.2025.16.2.2326
14. Fadhiliani, D., Umam, K., Suhartati, S., & Johar, R. (2025). Mathematical Modeling of Student Learning Outcomes using E-learning-Based Remedial Programs. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, *9*(3). https://doi.org/10.31764/jtam.v9i3.31404
15. Ilkiv, O., Matviyiv, V., & Yasinskyi, A. (2025). «Computer technology and mathematical statistics» as a means of forming innovative competence in higher education institutions of physical culture and sports. *Scientific Journal of National Pedagogical Dragomanov University. Series 15. Scientific and Pedagogical Problems of Physical Culture (Physical Culture and Sports)*, *5K(191)*. https://doi.org/10.31392/udu-nc.series15.2025.05k(191).18
16. Kalinin, S. I., & Toropova, S. I. (2025). Integration of computational thinking into the mathematical education of students of natural sciences. *Perspektivy Nauki i Obrazovania*, *74*(2). https://doi.org/10.32744/pse.2025.2.11

17. Kaminski, M. R., Boscarioli, C., & Klüber, T. E. (2025). Computational Thinking in Mathematical Modeling Practices from the Perspective of Meaningful Learning Theory. *Revista Internacional de Pesquisa Em Educação Matemática*, *15*(3). https://doi.org/10.37001/ripem.v15i3.4452
18. Klochko, V. I., Klochko, O. v., & Fedorets, V. M. (2025). Mathematics for digital signal processing: series and integrals of Fourier in professional training. *Journal of Physics: Conference Series*, *3105*(1). https://doi.org/10.1088/1742-6596/3105/1/012007
19. Kovtoniuk, M. M., Soia, O. M., Kosovets, O. P., & Tiutiun, L. A. (2025). Virtual educational mathematics park as a platform for educational and research activities at the university. *Journal of Physics: Conference Series*, *3105*(1). https://doi.org/10.1088/1742-6596/3105/1/012024
20. Laja, Y. P. W., Delvion, E. B. S., & Siahaan, M. M. L. (2025). Chatbot Artificial Intelligence (AI) to Supporting Differentiated Mathematics Learning. *Jurnal Pendidikan MIPA*, *26*(1). https://doi.org/10.23960/jpmipa.v26i1.pp118-130
21. Martins Bessa, S., Soares Oliveira, F. W., & Floriano Paulino, O. (2025). Mathematical modeling: a study on its incorporation into basic education from the proceedings of SIPEM. *Educação Matemática Pesquisa*, *27*(1).
22. Matorin, D. D., & Cherepkov, A. Yu. (2025). Numerical analysis of multicomponent dynamic models of stage-by-stage knowledge assimilation. *Nonlinear World*. https://doi.org/10.18127/j20700970-202501-04
23. Mu'arif, W., Mustaqimah, Prasetyawati, A., Suryani, Maryati, T., & Rokhimah, S. (2025). Penerapan Model Realistic Mathematic Education (RME) untuk Meningkatkan Hasil Belajar pada Pembelajaran Matematika. *Jurnal Educazione : Jurnal Pendidikan, Pembelajaran Dan Bimbingan Dan Konseling*, *2*(2).
24. Nyirenda, J. L. Z., Mbakaya, B. C., Wagner, D., Stete, K., Jaeger, V. K., Bartz, A., Karch, A., Nanyinza, E., Soko, M., Chimbatata, N., Chirambo, G. B., & Lange, B. (2025). NOZGEKA—a collaborative curriculum development process for a Master of Science in Public health with a focus on infectious diseases epidemiology in Malawi: a perspective. *Humanities and Social Sciences Communications*, *12*(1).

https://doi.org/10.1057/s41599-025-05654-6

25. Orazali, G., Dauletkulova, A., & Mekebayev, N. (2025). METHODOLOGY OF TEACHING GEOMETRY IN THE CONTEXT OF DIGITALIZATION OF EDUCATION. *Журнал Серии «Педагогические Науки»*, *76*(1). https://doi.org/10.48371/peds.2025.76.1.029
26. Pérez-Mojica, R., & Osorio-Gutiérrez, P. A. (2025). Diseño de actividades de modelación matemática para solucionar problemas reales de la función cuadrática. *Praxis*, *21*(2). https://doi.org/10.21676/23897856.6079
27. Podkhodova, N., Sheremetyeva, O., & Soldaeva, M. (2025). How to teach mathematics so that students truly understand it. *South Florida Journal of Development*, *6*(9). https://doi.org/10.46932/sfjdv6n9-001
28. Saeki, A., Kawakami, T., & Lamb, J. (2025). The role of children's analogies in the transfer of their knowledge in mathematical modelling processes. *ZDM - Mathematics Education*, *57*(2). https://doi.org/10.1007/s11858-025-01675-2
29. Sahani, S. K., & Karna, S. K. (2025). Program for Cultural Education Optimization by the Application of Numerical Methods. *Journal of Posthumanism*, *5*(1). https://doi.org/10.63332/joph.v5i1.2687
30. Sakibayeva, B., & Sakibayev, S. (2025). Enhancing professional competence in physics and mathematics education through information technology. *Educational Research and Evaluation*, *30*(7–8). https://doi.org/10.1080/13803611.2025.2502813
31. Sardor, S., & Uldona, S. (2025). THE IMPORTANCE OF STUDYING MATHEMATICS AND MODERN APPROACHES. *JournalNX- A Multidisciplinary Peer Reviewed Journal*, *11*.
32. Shcherbatykh, S. v., Lykova, K. G., Igonina, E. v., & Simonovskaya, G. A. (2025). Development of cognitive mobility when teaching probability theory and mathematical statistics to students using digital solutions. *Perspektivy Nauki i Obrazovania*, *74*(2). https://doi.org/10.32744/pse.2025.2.12
33. Smirnov, E. I., Skornyakova, A. Y. U., & Tikhomirov, S. A. (2025). The ontology and engineering of professional self-determination of school students in synergy

manifestation of natural science with mathematical and digital methods. *Perspektivy Nauki i Obrazovania*, *3*. https://doi.org/10.32744/pse.2025.3.36

34. Solod, L. V., Tkachova, V. V., Prokofieva, H. Ya., & Bereziuk, H. H. (2025). MODELING THE STUDY OF SOLID MATERIALS' THERMAL CONDUCTIVITY USING THE DYNAMIC CALORIMETRY METHOD. *Ukrainian Journal of Civil Engineering and Architecture*, *3 (027)*. https://doi.org/10.30838/ujcea.2312.270425.129.1169
35. Syifa Afidah Nurul Arifin. (2025). Systematic Literature Review: Kemampuan Pemodelan Matematika Siswa SMP. *Griya Journal of Mathematics Education and Application*, *5*(3). https://doi.org/10.29303/griya.v5i3.802
36. Ulker, D., & Ozaltun Celik, A. (2025). A qualitative metasynthesis: Interaction between mathematical modeling and higher order thinking. *Thinking Skills and Creativity*, *56*. https://doi.org/10.1016/j.tsc.2025.101773
37. Villarreal, M. E., & Coirini Carreras, A. (2025). Mathematical modeling in education: contributions and visibility of an Argentine research group. *ZDM - Mathematics Education*, *57*(7). https://doi.org/10.1007/s11858-025-01733-9
38. Villota-Cadena, Á. P., Sandoval-Palis, I. P., Grijalva-Villegas, G. F., & Herrera-Granda, E. P. (2025). CDR-Solv: Solving the Convection-Diffusion-Reaction Equation with Algebraic Sub-Grid Scale Stabilization Using Python. *Applied Sciences (Switzerland)*, *15*(18). https://doi.org/10.3390/app151810256
39. Zarubin, V. I., & Andreeva, I. v. (2025). Integrated process-project model of functioning of a secondary comprehensive educational organization. *Vestnik Majkopskogo Gosudarstvennogo Tehnologiceskogo Universiteta*, *2*. https://doi.org/10.47370/2078-1024-2025-17-2-81-98

# Chapter IV.

# Epistemological Convergence: The Integrated Role of Prototyping, Mathematical Modeling, and the Scientific Method in Complex Systems Engineering

## 1. The Teleological Nature of the Artifact

The contemporary engineering landscape is defined by a fundamental convergence between the abstract rigor of the scientific method and the pragmatic utility of artifact creation. Historically, science was perceived as the pursuit of knowledge—understanding "what is"—while engineering was viewed as the pursuit of utility—creating "what is not yet." However, the increasing complexity of modern systems, from the gravitational wave detectors of LIGO to the bio-fidelic digital twins of the human heart, has necessitated a unification of these domains. The act of engineering has become an experimental science, where the prototype serves as a physical hypothesis and the mathematical model acts as the theoretical framework governing that hypothesis.

This report explores the intricate relationship between prototyping, mathematical modeling, and the scientific method. It posits that the design process is, at its core, a rigorous exercise in hypothesis testing. Whether the "experiment" is a Monte Carlo simulation running on a supercomputer or a physical crash test of an automotive chassis, the epistemological goal remains the same: the reduction of uncertainty and the validation of predicted behavior against empirical reality. We must begin, however, with a philosophical caution rooted in the limitations of representation. As noted by S. I. Hayakawa and reiterated in the foundational texts of mathematical modeling, "The map is NOT the territory it stands for".[1] A model is an abstraction, a deliberate simplification of reality designed to answer specific questions.[2] The danger in modern R&D lies not in the use of models, but in the confusion of the symbol with the thing symbolized. Thus, the validation of models against the "territory" of physical reality remains the central challenge of systems engineering.

In examining this convergence, we must recognize the teleological nature of modeling and simulation. Unlike a work of art, which may exist for its own sake, a model is defined by its purpose. As defined in the literature, a model helps us to answer questions and solve problems.[2] Beginners in the field often fall into the trap of believing that a "good" model is one that mimics reality as closely as possible in every dimension. This is a fallacy. Modeling aims at simplification, not the useless production of complex copies of a complex reality.[2] The utility of a model is derived from its ability to strip away the noise of the real world to reveal the signal of the underlying mechanism. This report will traverse the theoretical foundations of this abstraction, the rigorous structures of the V-Model in systems engineering, the economic imperatives driving the shift from physical to virtual testing, and the detailed case studies that illustrate the triumph—and occasional failure—of this integrated methodology.

# 2. Epistemological Foundations: Engineering as Experimental Science

## 2.1 The Hypothesis of the Artifact

The scientific method, in its traditional formulation, involves a cycle of observation, hypothesis formulation, experimentation, and conclusion.[3] A scientist observes a phenomenon, formulates a hypothesis to explain it, and designs an experiment to falsify that hypothesis. If the experiment fails to refute the hypothesis, confidence in the theory grows.[5] In the context of engineering, the "hypothesis" takes a tangible form. The engineer hypothesizes that a specific configuration of materials, geometry, and logic will satisfy a set of functional requirements within a given set of constraints.[4] The prototype—whether it is a physical assembly of steel and silicon or a virtual assembly of code and equations—is the experiment designed to test this hypothesis.

There is a distinct parallel between the "Strong Inference" method in biology and the iterative design process in engineering. Strong Inference involves devising alternative hypotheses, devising a crucial experiment to exclude one or more of them, and carrying out the experiment to get a clean result.[6] Similarly, in engineering, prototyping is the mechanism for excluding design candidates that fail to meet performance criteria. When a prototype fails, it is an instance of epistemic refutation. It demonstrates that the engineer's understanding of the system dynamics, the material properties, or the environmental interactions was incomplete or incorrect.[7] Therefore, failure in prototyping is not merely an operational setback; it is a successful generation of new knowledge regarding the boundaries of the design space.[8]

The distinction between the scientific method and the engineering design process is often drawn in terms of their outputs: science produces knowledge, while engineering produces solutions. However, the *process* by which these outputs are achieved is increasingly identical.

- **Scientific Hypothesis:** "If X conditions are met, nature will behave in Y manner."
- **Engineering Hypothesis:** "If I construct X artifact, it will perform function Y under constraints Z."

In both cases, the rigorous testing of the hypothesis determines the validity of the work. The "experiment" in engineering is often a test of the prototype against the requirements defined in the design phase.[10] If the prototype functions as predicted, the hypothesis is corroborated. If it malfunctions, the hypothesis is falsified, and the engineer must return to the theoretical drawing board—the mathematical model—to adjust the parameters of the hypothesis.[1]

## 2.2 Deductive vs. Inductive Modeling Paradigms

Mathematical models serve as the formal language for these engineering hypotheses. They allow engineers to predict behavior before physical resources are committed, acting as a filter for ideas that are theoretically unsound. We can categorize these modeling approaches into two broad epistemological camps: deductive and inductive.

Deductive Modeling relies on a priori information, typically the fundamental laws of nature.[11] These are often referred to as "white-box" or "physics-based" models. They derive the behavior of a system from first principles—Newton's laws of motion, Maxwell's equations of electromagnetism, or the laws of thermodynamics.[13] The strength of deductive modeling lies in its universality and explainability. If a bridge collapses in a physics-based simulation, we can trace the failure to a specific stress concentration that exceeded the yield strength of the material. This allows for "Strong Inference" where the mechanism of failure is understood and can be corrected.

Inductive Modeling, by contrast, derives relationships from observed data. These are "black-box" or "data-driven" models, increasingly prevalent with the rise of machine learning and artificial intelligence.[13] In this paradigm, the model "learns" the behavior of the system by analyzing large datasets of inputs and outputs. While these models can be incredibly accurate, particularly for complex, chaotic, or poorly understood phenomena where the governing equations are intractable, they lack the explanatory power of deductive models. An inductive model might predict *that* a component will fail, but it may not be able to explain *why* in terms of fundamental physics.

The modern scientific method in engineering increasingly relies on a hybrid approach. Engineers use data-driven methods to calibrate the parameters of physics-based models, thereby refining the "map" to better fit the "territory".[13] This synthesis allows for the rigor of physical laws to be combined with the empirical precision of real-world data, creating a "Gray Box" model that is both explainable and accurate.

## 2.3 The Role of Failure in Epistemology

In the philosophy of science, particularly the Popperian view, the only relevant evidence is negative evidence. One can never prove a hypothesis is true; one can only fail to refute it.[5] This logical asymmetry is central to the engineering epistemology of prototyping. A successful prototype test does not prove that the design is perfect; it merely proves that it works under the specific conditions tested. A failed prototype test, however, provides absolute proof that the design (or the model used to create it) is flawed.

This perspective reframes "failure" from a negative outcome to a critical epistemic event. As noted in the literature on engineering epistemology, failure is an inevitable and essential part of physical prototype execution because it provides the feedback necessary to correct the mental and mathematical models of the engineer.[7] The "glitch" in the software or the fracture in the strut is the moment where reality asserts itself against the abstraction of the design. It is the "counter-example" that refutes the hypothesis.

However, epistemic failure can also occur when engineers believe something "that just ain't so".[14] This occurs when a model is relied upon outside of its domain of validity, or when "tribal knowledge" in an engineering organization goes unchallenged by empirical testing. The rigorous application of the scientific method—specifically the requirement for falsifiability—protects against this form of intellectual complacency. By treating every design decision as a hypothesis that must be tested, engineers can systematically reduce the "epistemic uncertainty" (lack of knowledge) in the system, leaving only the "aleatory uncertainty" (natural randomness) to be managed.[15]

# 3. The Mathematical Model: Abstraction, Formalism, and Classification

## 3.1 Taxonomy of Mathematical Models

To understand how models function as scientific tools within the engineering workflow, we must categorize them based on their mathematical structure and their relationship to time, certainty, and state. A rigorous taxonomy allows engineers to select the appropriate "tool" for the specific "experiment" they wish to conduct.

## 3.2 The Role of Simulation as Second-Order Experimentation

Simulation is the execution of a mathematical model over time. It has been described in the philosophy of science as a "second-order experiment".[16] If a thought experiment is a simulation run in the mind, and a physical experiment is a simulation run in matter, a computer simulation is an experiment run in logic. Simulation allows for the exploration of scenarios that are dangerous, expensive, or impossible to replicate physically—such as a nuclear core meltdown, the collision of galaxies, or the spread of a pandemic.[17]

A crucial technique in modern simulation is the Monte Carlo method, which relies on repeated random sampling to solve deterministic problems or model physical systems with significant uncertainty.[18] This method is particularly vital in fields like high-energy physics. In the context of the Large Hadron Collider (LHC), Monte Carlo simulations are used to generate billions of simulated proton-proton collisions. These simulations provide the "control group" for the experiment; they predict what the detectors *should* see if the Standard Model of particle physics is correct. When the real data collected by the detectors deviates from these Monte Carlo simulations, it indicates the potential discovery of new physics, such as the Higgs boson.[19]

## 3.3 Fidelity vs. Cost: The Trade-off Landscape

The pursuit of high fidelity in modeling comes at a significant cost—both in terms of computational resources and the time required to configure and run the simulation. This creates a fundamental trade-off that engineers must navigate.

- **High-Fidelity Models:** These models aim to capture the physics of the system with minimal abstraction. Examples include Large Eddy Simulation (LES) in computational fluid dynamics (CFD) or detailed finite element analysis (FEA) of a crash structure. They require massive computing power and are slow to solve, making them unsuitable for rapid iteration or real-time testing.[21]

- **Low-Fidelity Models:** These models use abstractions—such as "lumped parameters" or linearized equations—to reduce computational cost. They are fast enough to run in real-time, making them ideal for Hardware-in-the-Loop (HIL) testing where a physical controller must interact with the model at millisecond intervals.[21]

The choice of fidelity is an economic and epistemological decision. Engineers must "right-size" the model to the question at hand.[23] Using a high-fidelity model to answer a basic architectural question is a waste of resources (a violation of the "Occam's Razor" principle in engineering). Conversely, using a low-fidelity model to validate a safety-critical margin can lead to catastrophic failure if the abstractions mask critical non-linear behaviors.[24]

The energy consumption of modeling is also becoming a relevant factor. Higher fidelity models generally require larger datasets and more complex computations, leading to a larger environmental footprint.[22] This sustainability constraint adds another dimension to the trade-off matrix, pushing the industry toward more efficient algorithms and "reduced order modeling" techniques that seek to retain accuracy while slashing computational cost.

# 4. The Architecture of the Prototype: Physical vs. Virtual

## 4.1 From CAD to Virtual Prototyping (VP)

Virtual Prototyping (VP) represents the evolution of Computer-Aided Design (CAD) from a static drafting tool to a dynamic testing environment. VP allows engineers to simulate the kinematics, dynamics, and control systems of a product in a purely digital environment before any physical material is cut.[25] This shift is driven by the desire to "shift left"—to move the validation and testing phases earlier in the design cycle where the cost of making changes is orders of magnitude lower.

In the automotive industry, VP has become sophisticated enough to replace physical prototypes for certain validation steps, such as component interference checking, basic aerodynamic profiling, and ergonomic assessments.[25] The "Zero Prototype" ambition—the goal of going directly from simulation to production—is now a stated objective for some manufacturers, particularly in the context of "Zero Prototypes Summits" hosted by simulation companies like VI-grade.[25] However, this ambition is tempered by the reality that virtual models, no matter how advanced, are still approximations of reality.

## 4.2 The Digital Twin Paradigm

The concept of the Digital Twin (DT) goes beyond a mere simulation or a virtual prototype. It implies a persistent, data-driven connection between the virtual model and a specific physical asset. The IEEE and industrial literature provide a rigorous taxonomy for this concept, distinguishing between three key states of the Digital Twin [27]:

1. **Digital Twin Prototype (DTP):** This is the DT that exists *before* the physical product is built. It contains the essential information to create the physical asset—the Bill of Materials (BOM), the 3D CAD models, the control code, and the manufacturing instructions. The DTP serves as the "hypothesis" of the product. It allows for virtual testing and optimization of the design before capital is committed to production tooling.
2. **Digital Twin Instance (DTI):** This is the specific virtual counterpart of an individual physical asset, created once the product is manufactured. The DTI remains linked to its physical twin throughout its lifecycle, receiving real-time data from sensors. It captures the unique history of that specific unit—its operational hours, the stress loads it has endured, and its maintenance history. The DTI is used for predictive maintenance and condition monitoring.[27]

3. **Digital Twin Environment (DTE):** This is the integrated platform where multiple DTIs and DTPs operate, allowing for system-of-systems analysis.[28]

Leading technology providers like AWS (with IoT TwinMaker) and IBM have developed platforms to operationalize these concepts. For instance, Carrier uses AWS to create digital twins of its cold chain solutions, allowing for the rapid development and testing of new refrigeration systems in a virtual environment.[29] Similarly, INVISTA (a Koch Industries subsidiary) uses digital twins of its manufacturing operations to give staff a complete digital view of assets and data, enabling optimization of the production process.[29] These implementations typically follow a workflow of data collection, virtual modeling, live data integration, and finally, analysis and simulation.[30]

## 4.3 Hardware-in-the-Loop (HIL)

Bridging the gap between pure simulation (Virtual Prototyping) and physical testing is Hardware-in-the-Loop (HIL) simulation. In an HIL setup, a physical controller (e.g., the Engine Control Unit of a car or the flight computer of a drone) is connected to a real-time simulator that models the behavior of the plant (the engine or the aircraft).[31]

HIL is a critical "integration" step in the V-Model. It allows engineers to validate the software and control logic without risking the expensive physical plant. For example, testing a battery management system (BMS) for an electric vehicle using HIL allows engineers to simulate extreme conditions—such as a thermal runaway or a short circuit—that would be dangerous and destructive to test with a real battery pack.[33] The simulator tricks the physical controller into believing it is connected to a real battery, allowing for rigorous testing of the safety logic.

The utility of HIL extends to the power grid as well. Researchers at the **DLR Grid Lab** use HIL to emulate grid networks, allowing them to test how renewable energy inverters interact with the power system without risking stability of the actual grid.[32] This capability is essential for validating the "smart" components of the modern energy infrastructure, where the behavior of software-defined inverters can be radically different from traditional rotating generators.[33]

# 5. Systems Engineering Frameworks: The V-Model and Validation

## 5.1 Structure of the V-Model

The management of complexity in modern engineering requires a structured approach to ensure that the "scientific method" is applied consistently across thousands of requirements and components. The V-Model is the standard framework for this process. It bends the linear "Waterfall" model into a V-shape to emphasize the direct relationship between the design phases (on the left) and the testing phases (on the right).[34]The V-Model ensures that every "hypothesis" generated on the left side (e.g., "The system shall stop within 50 meters") has a corresponding "experiment" on the right side (e.g., "Brake Test A"). This structure prevents the "Big Bang" testing approach, where all testing is left until the end, often resulting in catastrophic discovery of deep-seated architectural flaws.[36]

## 5.2 Verification vs. Validation: The Epistemic Distinction

A critical distinction in systems engineering, often confused by laypeople, is the difference between Verification and Validation (V&V). This distinction is vital for understanding the scientific application of engineering.[15]

- **Verification:** "Are we building the product *right*?" This is a check against the specifications. It asks whether the system meets the requirements defined in the previous step. Verification is often an internal mathematical or logical check—for example, verifying that the code compiles without errors or that the stress analysis shows a safety factor of 2.0 as required.[15]
- **Validation:** "Are we building the *right* product?" This is a check against the user's needs and the real world. It asks whether the system, even if it meets all specifications,

actually solves the problem it was intended to solve. Validation requires external confirmation, often through physical prototyping or field trials. A product can be fully verified (it meets every spec) but fail validation (users hate it or it doesn't work in the actual environment).[17]

In the context of mathematical modeling, Code Verification ensures the equations are solved correctly by the computer (the math is right), while Model Validation ensures the equations accurately represent the physical reality (the physics is right).[15] The ASME V&V 40 standard provides a risk-informed framework for assessing the credibility of computational models in medical device development, illustrating how these concepts are formalized in regulated industries.[40]

# 6. Economic Imperatives: The Cost of Change and ROI

## 6.1 The 1-10-100 Rule and the Cost of Change Curve

The drive toward mathematical modeling, virtual prototyping, and the rigorous application of the V-Model is not merely an academic exercise; it is fundamentally driven by economic imperatives. The Cost of Change curve, often formalized as the 1-10-100 Rule, dictates that the cost of fixing a defect increases exponentially as the product moves through its lifecycle.[41]

- **Concept/Simulation Phase (1):** Identifying an error here involves changing a line of code or a parameter in a model. The cost is negligible—time and electricity.
- **Design/Drafting Phase (10):** Fixing an error here requires updating CAD drawings, reviewing BOMs, and potentially re-doing some analysis. The cost is administrative and labor-intensive.
- **Production Phase (100):** Fixing an error here means scrapping physical tooling, discarding manufactured parts, and halting the production line. The cost is material and operational.
- **Field/Operation Phase (1,000+):** Fixing an error after the product is released leads to

recalls, warranty claims, lawsuits, and brand damage. The cost can be existential for the company.[42]

This economic reality forces engineers to "shift left"—to move the discovery of defects as early in the process as possible. Virtual prototyping allows engineers to iterate hundreds of times at the 1 level (Simulation), whereas physical prototyping often forces discovery at the 100 level or higher.

## 6.2 The ROI of Virtualization: Boeing 777 and Beyond

The aerospace industry provides a potent example of the Return on Investment (ROI) for virtual prototyping. The Boeing 777 was the first commercial aircraft designed entirely using 3D CAD (CATIA), eliminating the need for a massive, full-scale physical "iron bird" mock-up that was traditionally used to check for cable and piping interference.[44]

- **The Savings:** By trusting the geometric model (the Digital Twin Prototype), Boeing reduced rework and "fit-up" errors during assembly. The model *was* the master reference. This "digital pre-assembly" saved Boeing an estimated 90% of their pre-production time for certain processes.[45]
- **Data-Driven Assembly:** Modern iterations of this process use data from past assemblies (e.g., 10,076 laser gap measurements) to train PCA (Principal Component Analysis) models. These models predict "shim gaps" for new aircraft, allowing parts to be machined to the correct tolerance *before* they are brought to the assembly line.[44] This is a move from deterministic modeling (nominal CAD) to statistical modeling (predictive assembly).

## 6.3 The Economics of Crash Testing

In the automotive sector, the cost differential between physical and virtual testing is stark. A physical crash test destroys a prototype vehicle that can cost hundreds of thousands of dollars to hand-build. A virtual crash test using Finite Element Analysis (FEA) costs only the computing time and the software license.

- **Scale:** Virtual testing allows for the evaluation of thousands of impact angles and speeds, whereas physical testing is limited to a handful of regulatory scenarios.[46]
- **Accuracy:** While virtual testing is cheaper, it must be accurate to be valuable. Studies comparing physical vs. virtual prototypes have shown that while virtual models are improving, physical models still hold the edge in certain domains. For example, a study on ship bridge workstations found that physical models yielded **96% accuracy** in material estimation, while virtual models only achieved **76%**.[48]
- **Sensory Validation:** The same study noted that physical prototypes elicited more positive emotional responses (valence) from users. This highlights a limitation of the virtual ROI calculation: it often fails to account for the "subjective" quality of the design—how it *feels* to the user—which is difficult to quantify in a digital model.[48]

# 7. Deep Dive Case Studies: The Scientific Method in Action

To fully understand the interplay of prototyping, modeling, and the scientific method, we must examine their application in specific, high-stakes domains where the failure of the "hypothesis" has profound consequences.

## 7.1 Case Study: High Energy Physics & Astrophysics (LHC & LIGO)

Fields dealing with the fundamental laws of nature often operate at scales—subatomic or cosmic—where direct physical prototyping is impossible or prohibitively expensive.

The Large Hadron Collider (LHC):
The ATLAS and CMS experiments at CERN rely on a complete digital replication of the detectors to function.

- **The Simulation:** The Geant4 toolkit is used to simulate the interaction of particles with the matter of the detector. This is a "Monte Carlo" simulation because particle decay is probabilistic.[19]

- **The Workflow:** Before the detector was built, simulations predicted its response to ensure the design was valid. Now, during operation, "real" data is compared against "simulated" data. If the real data deviates from the simulation (which represents the "Standard Model" background), it indicates the potential existence of new particles. The simulation is the "control," and the physical detector is the "variable".[20]
- **Resource Constraints:** The simulation is so computationally expensive that it consumes the majority of the Worldwide LHC Computing Grid's resources. Researchers are now exploring "fast simulation" techniques (using Generative Adversarial Networks - GANs) to approximate the Geant4 results, trading fidelity for speed—a classic modeling trade-off.[19]

LIGO (Laser Interferometer Gravitational-Wave Observatory):

LIGO detects ripples in spacetime using mirrors suspended by multi-stage pendulums. The isolation requirements are so extreme that physical prototyping is difficult due to environmental noise.

- **Mathematica vs. MATLAB:** Researchers utilized Mathematica for the exact analytical derivation of the equations of motion (symbolic modeling) to understand the physics deeply. They then used MATLAB for numerical state-space analysis to design the control systems.[51] This shows the complementary use of different modeling tools for different epistemological needs (understanding vs. control).
- **The Prototype Refutation:** A physical prototype of the "double pendulum" suspension was built at MIT to verify the models. The mathematical model initially failed to predict specific **"cross-coupling"** effects (e.g., longitudinal motion causing pitch rotation). The physical experiment revealed "unmodeled dynamics"—specifically regarding the wire attachment points and the flexure stiffness of the blades.[52]
- **The Loop:** The experimental data from the physical prototype was used to update the mathematical model (adding "h" parameters for attachment point offsets). The refined model was then sufficiently accurate to design the final "Quadruple Pendulum" for

Advanced LIGO without building a full-scale intermediate prototype for every iteration.[52]

- **Insight:** This perfectly illustrates the scientific method in engineering. The Model was the Hypothesis. The Physical Prototype was the Experiment. The discrepancy (refutation) led to a Theory Update (better model), which enabled the Final Design.

## 7.2 Case Study: Biomedical Engineering (In Silico Trials & The Ventilator Splitter)

Highly variable biological systems where "standardization" is impossible and human life is at risk.

In Silico Clinical Trials (ISCT):

Testing medical devices (e.g., stents, heart valves) typically requires animal and human trials, which are slow, expensive, and ethically complex. ISCT uses computational cohorts—"virtual patients"—to test devices before they touch a human.54

- **Regulatory Credibility:** The FDA and the **ASME V&V 40** standard have established a framework for determining the "credibility" of a model. This is a legal and scientific acceptance that a simulation can, in specific contexts, stand in for a human life.[40]
- **The Digital Heart:** Researchers are creating Digital Twins of the human heart, modeling electrophysiology (calcium ion flow) and hemodynamics.[56] A study showed that a digital twin of the pulmonary artery could predict pressure with sufficient accuracy to replace invasive catheterization in heart failure management.[56] However, these models must be calibrated carefully; assuming "well-mixed" compartments for drugs can lead to significant errors if mass-transfer limitations in tissue are ignored.[24]

The Failure of the Ventilator Splitter:

During the COVID-19 pandemic, a task force attempted to design a ventilator splitter to allow one machine to serve multiple patients.

- **The Modeling Success (Apparent):** Using **Computational Fluid Dynamics (CFD)**, engineers optimized the valve geometry to regulate airflow to 30%, 50%, 70%, etc. The model predicted success.[58]

- **The Prototyping Failure:** When 3D-printed prototypes were tested at Memorial Hospital with actual ventilators, the splitter failed to modify airflow enough to match the CFD predictions.[58]
- **The Cause:** The boundary conditions of the CFD model likely did not match the complex, dynamic pressure response of the actual ventilator hardware and the patient's lung compliance.
- **The Outcome:** The project was closed. This is a stark reminder that a verified model (mathematically correct) may not be a validated model (physically correct). The physical prototype was the ultimate arbiter of truth.

# 8. Physical Prototyping in the Digital Age: Why We Still Build

Despite the power of the Digital Twin and the success of "Zero Prototype" initiatives in specific domains, physical prototyping remains an essential component of the scientific method in engineering. The reasons for this are epistemological, sensory, and practical.

1. **Unmodeled Dynamics (The "Unknown Unknowns"):** Mathematical models only contain the physics we *know* to include. Reality contains *all* physics, including the ones we forgot, ignored, or don't strictly understand (e.g., the wire flexure in LIGO).[52] The physical prototype is the only "perfect" simulation of reality because it *is* reality. It integrates all physical laws—thermal, electromagnetic, mechanical, chemical—simultaneously and without abstraction.
2. **Epistemic Opacity:** As simulations become more complex (e.g., Deep Learning models or massive multi-physics simulations), they become "black boxes." We know *that* they predict the output, but we may not understand *why*. Physical testing provides a "ground truth" that cuts through this algorithmic opacity.[13]

3. **Human-Centric Validation (Sensory & Emotional):** Haptic feedback, aesthetics, and ergonomic comfort are difficult to simulate. As shown in the comparison study, physical prototypes significantly outperform virtual ones in evaluating "emotional" and "sensory" requirements.[48] You cannot touch a digital steering wheel to feel the grain of the leather or the damping of the switch.
4. **Integration Chaos:** Components may work individually in simulation, but when physically connected, manufacturing tolerances, thermal expansion, and electromagnetic interference can cause system-level failure. The physical prototype exposes these interface failures.[59]

# 9. The Hybrid Epistemology

The integration of prototyping, mathematical modeling, and the scientific method has created a new epistemology for engineering—one that is hybrid, recursive, and risk-aware. We no longer view the "scientific method" as the exclusive preserve of natural philosophers, nor "prototyping" as the exclusive domain of the workshop artisan. Instead, we see a unified workflow where the digital and the physical are in constant dialogue.

1. **Observation:** Data is collected from the physical world (via sensors or historical data).
2. **Hypothesis (Model/DTP):** A mathematical representation is constructed to explain this data and predict future behavior.
3. **Experiment (Simulation):** The model is stressed in the virtual domain (Monte Carlo, FEA) to filter out poor designs (the 1 cost of change).
4. **Refinement:** The design is optimized based on virtual feedback.
5. **Validation (Physical Prototype):** The optimized design is instantiated in matter. This is the critical test.
6. **Refutation or Corroboration:** If the physical prototype fails (like the ventilator splitter), the model is refuted and must be updated (as in LIGO). If it succeeds (like the Boeing 777 fit-up), the design is validated.
7. **Operation (Digital Twin Instance):** The physical and virtual assets continue to exist in

a feedback loop, where the physical provides data and the virtual provides insight for maintenance and optimization.

In this framework, the "failure" of a prototype is not an error; it is a necessary step in the calibration of our understanding. It is the moment where the map is forced to align with the territory. As we move toward autonomous experimentation and AI-driven design, this grounding in the scientific method—the commitment to validating the abstract against the concrete—will remain the essential safeguard against the hubris of the simulation. The engineer of the future is neither just a builder nor just a mathematician, but an experimentalist operating at the interface of the virtual and the real, using every tool available to rigorously test the hypothesis of the artifact.

# Chapter bibliography


1. Gokulakrishnan. J., & C. Gantham. P. (2024). Ergonomics Evaluation an Origin and Overview - Copy. *International Journal For Multidisciplinary Research*, *6*(2). https://doi.org/10.36948/ijfmr.2024.v06i02.15960
2. Ahmad Laskar, A., Ahmed, M., Vo, D. V. N., Abdullah, A., Shahadat, M., Mahmoud, M. H., Khan, W., & Yusuf, M. (2023). Mathematical modeling and regression analysis using MATLAB for optimization of microwave drying efficiency of banana. *Thermal Science and Engineering Progress*, *46*. https://doi.org/10.1016/j.tsep.2023.102157
3. Al-Ezzi, A. S., & Ansari, M. N. M. (2022). Photovoltaic Solar Cells: A Review. In *Applied System Innovation* (Vol. 5, Issue 4). https://doi.org/10.3390/asi5040067
4. Bardini, R., & di Carlo, S. (2024). Computational methods for biofabrication in tissue engineering and regenerative medicine - a literature review. In *Computational and Structural Biotechnology Journal* (Vol. 23). https://doi.org/10.1016/j.csbj.2023.12.035
5. Çevik, M., Savaşaneril, N. B., & Sezer, M. (2025). A Review of Polynomial Matrix Collocation Methods in Engineering and Scientific Applications. In *Archives of Computational Methods in Engineering* (Vol. 32, Issue 6). https://doi.org/10.1007/s11831-025-10235-6
6. Dond, D. K., Gulhane, N. P., & Dhamejani, C. (2018). Mathematical Modelling & MATLAB Simulation of Diesel Engine. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3101296
7. Faroughi, S. A., Pawar, N. M., Fernandes, C., Raissi, M., Das, S., Kalantari, N. K., & Mahjour, S. K. (2024). Physics-Guided, Physics- Informed, and Physics-Encoded Neural Networks and Operators in Scientific Computing: Fluid and Solid Mechanics. *Journal of Computing and Information Science in Engineering*, *24*(4). https://doi.org/10.1115/1.4064449
8. Iacovidou, E., Millward-Hopkins, J., Busch, J., Purnell, P., Velis, C. A., Hahladakis, J. N., Zwirner, O., & Brown, A. (2017). A pathway to circular economy: Developing a conceptual framework for complex value assessment of resources recovered from

waste. *Journal of Cleaner Production*, *168*. https://doi.org/10.1016/j.jclepro.2017.09.002

9. Idoko Peter Idoko, Gerald Chekwube Ezeamii, Christian Idogho, Enemali Peter, Ubong Sunday Obot, & Vitalis Afebuame Iguoba. (2024). Mathematical modeling and simulations using software like MATLAB, COMSOL and Python. *Magna Scientia Advanced Research and Reviews*, *12*(2). https://doi.org/10.30574/msarr.2024.12.2.0181
10. Jadhav, D., & Thakare, M. (2022). Mathematical Modeling and Simulation of a Fuel Cell in MATLAB. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4056056
11. Kurasov, D. (2020). Mathematical modeling system MatLab. *Journal of Physics: Conference Series*, *1691*(1). https://doi.org/10.1088/1742-6596/1691/1/012123
12. Lin, D. (2024). Application of Mathematical Modeling Based on MATLAB in Experimental Data Analysis. *Procedia Computer Science*, *247*(C). https://doi.org/10.1016/j.procs.2024.10.011
13. Ma, H., Li, L., Ma, Y., Liu, X., & Yang, S. (2025). Exploration and Application of Project-Based Teaching Integrating Scientific Research Achievements in Electronic Comprehensive Design Course. *Computer Applications in Engineering Education*, *33*(3). https://doi.org/10.1002/cae.70047
14. Martins, A. C. R. (2025). Human flaws and logical tools in engineering. *Canadian Journal of Chemical Engineering*, *103*(8). https://doi.org/10.1002/cjce.25764
15. Nevid, J. S., Pavlov, I., Powerlecture, F., Desktop, F., Weber, M. S., Jin, Z., Psy, S. I., Psy, F., Psychology, C., Fa, P. S. Y., Fa, P. S. Y., Reports, A., Action, T., Report, T. B., Stowell, J. R., Kim, O., Kendeou, P., van den Broek, P. P. D. M., White, M. J., … McNamara, D. S. (2011). An introduction to the psych package: Part II. Scale construction and psychometrics. *Science Education*, *1*(1). https://doi.org/10.1017/CBO9781107415324.004
16. Nguyen, X. H., & Nguyen, M. P. (2015). Mathematical modeling of photovoltaic cell/module/arrays with tags in Matlab/Simulink. *Environmental Systems Research*, *4*(1). https://doi.org/10.1186/s40068-015-0047-9
17. Orlova, E. v. (2022). Design Technology and AI-Based Decision Making Model for Digital Twin Engineering. *Future Internet*, *14*(9). https://doi.org/10.3390/fi14090248

18. Patriarca, R., di Gravio, G., Woltjer, R., Costantino, F., Praetorius, G., Ferreira, P., & Hollnagel, E. (2020). Framing the FRAM: A literature review on the functional resonance analysis method. *Safety Science*, *129*. https://doi.org/10.1016/j.ssci.2020.104827
19. Payette, M., & Abdul-Nour, G. (2023). Asset Management, Reliability and Prognostics Modeling Techniques. *Sustainability (Switzerland)*, *15*(9). https://doi.org/10.3390/su15097493
20. Peters, T. (2019). Data-driven science and engineering: machine learning, dynamical systems, and control. *Contemporary Physics*, *60*(4). https://doi.org/10.1080/00107514.2019.1665103
21. Pop, T., Buzduga, C., Pentiuc, R.-D., Ifrim, V.-C., & Ungureanu, C. (2022). Mathematical modeling in MATLAB of a photovoltaic panel. *IOP Conference Series: Materials Science and Engineering*, *1256*(1). https://doi.org/10.1088/1757-899x/1256/1/012037
22. Sobran, N. M. M., & Ismail, Z. H. (2023). A Systematic Literature Review of Unsupervised Fault Detection Approach for Complex Engineering System. *Journal of Advanced Research in Applied Mechanics*, *103*(1). https://doi.org/10.37934/aram.103.1.4360
23. Testov, V. A., & Perminov, E. A. (2023). TRANSDISCIPLINARY ROLE OF PHYSICAL AND MATHEMATICAL DISCIPLINES IN MODERN NATURAL SCIENCE AND ENGINEERING EDUCATION. *Obrazovanie i Nauka*, *25*(7). https://doi.org/10.17853/1994-5639-2023-7-14-43
24. Tian, Y., Chao, M. A., Kulkarni, C., Goebel, K., & Fink, O. (2022). Real-time model calibration with deep reinforcement learning. *Mechanical Systems and Signal Processing*, *165*. https://doi.org/10.1016/j.ymssp.2021.108284
25. Valkova-Jarvis, Z., Nenova, M., & Mihaylova, D. (2025). Hypercomplex Numbers—A Tool for Enhanced Efficiency and Intelligence in Digital Signal Processing. *Mathematics*, *13*(3). https://doi.org/10.3390/math13030504
26. Varenne, F. (2003b). La simulation informatique face à la « méthode des modèles ».

Le cas de la croissance des plantesThe computer simulation facing the “Method of Models”. The case of plant growth. *Nature Sciences Sociétés*, *11*(1). https://doi.org/10.1016/s1240-1307(03)00004-9

27. Wei, G., Zheng, B., Dong, J., Yang, Y., Yang, G., Song, S., Guo, S., & Qi, S. (2025). Research Progress of Three-Dimensional Engineering Geological Evaluation Modeling. In *Sustainability (Switzerland)* (Vol. 17, Issue 8). https://doi.org/10.3390/su17083739
28. Yu, R., & Wang, R. (2024). Learning dynamical systems from data: An introduction to physics-guided deep learning. *Proceedings of the National Academy of Sciences of the United States of America*, *121*(27). https://doi.org/10.1073/pnas.2311808121
29. Zhao, X. W., Tong, X. M., Ning, F. W., Cai, M. L., Han, F., & Li, H. G. (2025). Review of empowering computer-aided engineering with artificial intelligence. *Advances in Manufacturing*. https://doi.org/10.1007/s40436-025-00545-0
30. Zhu, C. (2020). Research on mathematical modeling strategy based on MATLAB from the perspective of innovative ability training. *Journal of Physics: Conference Series*, *1648*(2). https://doi.org/10.1088/1742-6596/1648/2/022017
31. Zhu, T., Wu, C., Zhang, Z., Li, Y., & Wu, T. (2025). Research on Evaluation Methods of Complex Product Design Based on Hybrid Kansei Engineering Modeling. *Symmetry*, *17*(2). https://doi.org/10.3390/sym17020306
32. Arayici, Y., Hamilton, A., & Gamito, P. (2006). Modelling 3D scanned data to visualise and analyse the Built Environment for regeneration. *Surveying and Built Environment*, *17*(2).
33. Bondada, B. R., Dwa, S., Magar, U. T., Denning, J., Tania, M., & Fails, J. A. (2025). Soul Support: Designing Hopeful Wearables for Children’s Emotional Wellness. *Proceedings - 24th Annual ACM Interaction Design and Children Conference, IDC 2025*. https://doi.org/10.1145/3713043.3737389
34. Emberley, A. C., & Moore, T. J. (2023). Professor Talk in Undergraduate, Introductory Design: A Multiple Case Study from Mechanical and Biomedical Engineering. *Studies in Engineering Education*, *4*(2). https://doi.org/10.21061/see.74

35. Gaudette, G., & Kafel, D. (2012). A Case Study: Data Management in Biomedical Engineering. *Journal of eScience Librarianship*, *1*(3). https://doi.org/10.7191/jeslib.2012.1027

36. Gerdsri, N., Kongthon, A., & Puengrusme, S. (2017). Profiling the Research Landscape in Emerging Areas Using Bibliometrics and Text Mining: A Case Study of Biomedical Engineering (BME) in Thailand. *International Journal of Innovation and Technology Management*, *14*(2). https://doi.org/10.1142/S0219877017400119

37. Hemsath, T. L. (2012). Using prototyping to teach digital fabrication techniques. *Electronic Journal of Information Technology in Construction*, *17*.

38. Kuroda, T. (2024). Slashing IC Power and Democratizing IC Access for the Digital Age. *IPSJ Transactions on System LSI Design Methodology*, *17*. https://doi.org/10.2197/ipsjtsldm.17.2

39. Montesinos, L., Santos-Diaz, A., Salinas-Navarro, D. E., & Cendejas-Zaragoza, L. (2022). Experiential Learning in Biomedical Engineering Education Using Wearable Devices: A Case Study in a Biomedical Signals and Systems Analysis Course. *Education Sciences*, *12*(9). https://doi.org/10.3390/educsci12090598

40. Moralioglu, B., & Gül, L. F. (2026). Envisioning tomorrow's spaces: A design fiction framework for exploring the integration of spatial computing into future architecture. *Futures*, *175*. https://doi.org/10.1016/j.futures.2025.103723

41. Pasupuleti, M. K. (2024). Revolutionizing Industries with Digital Twin Technology. In *Digital Twin Technology*. https://doi.org/10.62311/nesx/97806

42. Paternò, L., Dhokia, V., Menciassi, A., Bilzon, J., & Seminati, E. (2023). Correction: A personalised prosthetic liner with embedded sensor technology: a case study (BioMedical Engineering OnLine, (2020), 19, 1, (71), 10.1186/s12938-020-00814-y). In *BioMedical Engineering Online* (Vol. 22, Issue 1). https://doi.org/10.1186/s12938-023-01154-3

43. Saba, T., Saad, K. S. K., & Rashid, A. bin. (2024). Precise surface engineering: Leveraging chemical vapor deposition for enhanced biocompatibility and durability

in biomedical implants. In *Heliyon* (Vol. 10, Issue 18). https://doi.org/10.1016/j.heliyon.2024.e37976

44. Singh, B., & Nayyar, A. (2024). Exploring diverse use cases of digital twins projecting digital transformation: Unlocking potential, addressing challenges and viable solutions. In *Digital Twins for Smart Cities and Villages*. https://doi.org/10.1016/B978-0-443-28884-5.00027-0
45. Taylor, J. R., Milne, A. J., & Macritchie, J. (2023). New musical interfaces for older adults in residential care: assessing a user-centred design approach. *Disability and Rehabilitation: Assistive Technology*, *18*(5). https://doi.org/10.1080/17483107.2021.1881172
46. Videla-Reyes, R. (2023). CPA Methodology: educational technological design proposal to solve problems. *Pacific Journal of Technology Enhanced Learning*, *5*(1). https://doi.org/10.24135/pjtel.v5i1.169
47. Asamoah, G., Baah-Adade, A. K., & Ayikpah, Y. A. A. (2021). Effects of Emotional Intelligence Variables on Investor Behaviour in Ghana. In *Available Journal of Technology, Engineering, Management & Computational Sciences ISSN* (Vol. 1, Issue 1).
48. Gorard, S., Siddiqui, N., & See, B. H. (2015). Education Endowment Foundation Durham Shared Maths Project Evaluation report and Executive summary. *Review of Educational Research*, *1*(2).
49. Khairulin, O. (2023). Ontological modeling of the psychological field of a play. *Psihologìâ ì Suspìlʹstvo*, *2*(2023). https://doi.org/10.35774/pis2023.02.106
50. Nantwi2, S. K. N. and W. K. (2013). British Journal of Education, Learning and Development Psychology. *Jurnal EMBA*, *1*(4).
51. Nilsson, F., & Dunin-Woyseth, H. (2012). Building a culture of doctoral scholarship in architecture and design. A Belgian-Scandinavian case. *Nordic Journal of Architectural Research*, *23*(1).
52. Opderbeck, D. W. (2023). The End of the Law? Law, Theology, and Neuroscience. *Perspectives on Science and Christian Faith*, *75*(1). https://doi.org/10.56315/pscf3-23opderbeck

53. Paraponaris, C., Ermine, J.-L., Guittard, C., & Lièvre, P. (2012). Knowledge management in a French research community: a case study of GeCSO congress: Very Informal Newsletter on Library Automation. *VINE*, *42*(3/4).

54. Picciotto, R. (2018). Evaluation for an Open Society: Then and Now. *Journal of MultiDisciplinary Evaluation*, *14*(31). https://doi.org/10.56645/jmde.v14i31.503

55. Prem, E. (2015). The role of technology for innovation management research methodology. *ISPIM Conference Proceedings*, *June*.

56. Tkaczyk, V. (2018). Whose larynx is it? Fields of scholarly competence around 1900. *History of Humanities*, *3*(1). https://doi.org/10.1086/696302

57. Wang, C. S., Ma, Y. S., Peng, P. A., Zou, C. N., Xie, S. C., Xiao, W. J., Zhang, S. C., Hu, X. M., Wang, J., Gao, S., Hou, M. C., Zhu, X. M., Shao, L. Y., Wu, H. C., Liu, Z. F., Chen, Z. Q., Zhu, R. K., & Chen, X. (2025). A Century Journey from Sedimentology to Sedimentosphere Science: Review and Perspective. *Acta Sedimentologica Sinica*, *43*(5). https://doi.org/10.14027/j.issn.1000-0550.2025.037

58. Silander, M., Grindal, T., Gerard, S. N., & Salone, T. (2025). Learning Science and Engineering From Videos and Games: A Randomized Trial of PBS KIDS The Cat in the Hat Knows a Lot About That! *Educational Researcher*, *54*(6). https://doi.org/10.3102/0013189X251327187

59. Yu, K., Yao, Y., Zhang, W., Lu, L., Gao, Q., Zhang, P., & Sing, S. L. (2025). Defects, monitoring, and AI-enabled control in soft material additive manufacturing: a review. *Virtual and Physical Prototyping*, *20*(1). https://doi.org/10.1080/17452759.2025.2588456

# Conclusion

At the end of this review of the general methodological proposals for research in Computer Science, it is evident that the historical dichotomy between theory and practice, or between engineering and science, is an increasingly blurred and less useful distinction. We have found that rigorous computational research does not lie in the exclusive choice of a single tool, but in the intelligent orchestration of prototyping, mathematical modeling, and the scientific method.

The main lesson that emerges from the previous chapters is that these three methodologies form a continuous, virtuous and necessary feedback loop:

- *From Model to Prototype:* We saw that mathematical modeling provides us with the formal language to define the problem and the theoretical guarantees of the solution. However, a model without implementation runs the risk of remaining a sterile abstraction.
- *From Prototype to Evidence:* Prototyping acts as the bridge to reality, exposing our ideas to physical, hardware, and usability limitations. But a prototype without evaluation is simply a technical demonstration, not science.
- *From Evidence to Knowledge:* This is where the scientific method closes the circle. Through controlled experimentation and statistical analysis, we transform the observed behavior of the prototype into data, and that data into validated knowledge that refines, in turn, our initial mathematical models.

This methodological integration is more urgent today than ever. We are facing an era where software systems (such as deep neural networks or planetscale distributed systems) exhibit emergent behaviors that cannot always be fully deduced from first mathematical principles.

In this context, the researcher must be able to operate as a mathematician to formalize uncertainty, as an engineer to build the testing tools, and as a natural scientist to observe and explain digital phenomena that often operate as "black boxes." The renunciation of any of these three pillars weakens the ability of the discipline to explain the *why* and *how* of technological advances.

Research in Computer Science has ceased to be a purely artisanal activity to become a mature science with its own epistemological standards. This book has sought to equip the researcher with the confidence to navigate these standards.

We hope that the reader will close this volume not only with new techniques in his arsenal, but with a new perspective: the conviction that the best research is the one that dares to build to understand, that calculates to predict and that experiments to verify. The future of computing will depend on researchers who are not afraid to get their hands dirty with code or tire their minds with abstraction, understanding that true scientific progress lies at that intersection.

# Bibliography


Abbasabadi, N., & Ashayeri, M. (2024). Artificial Intelligence in Performance-Driven Design: Theories, Methods, and Tools. In *Artificial Intelligence in Performance-Driven Design: Theories, Methods, and Tools*. https://doi.org/10.1002/9781394172092

Abla, D. E. (2015). Agro-Input Use in Peri-Urban Production in the Greater Accra Region. *(Doctoral Dissertation, University of Ghana).*

Adhikari, M. S., Majumder, A., Sindhwani, M., & Sachdeva, S. (2025). AI and IoT integration for smart agriculture. In *Smart Electronic Devices: Artificial Intelligence, Machine Learning, and the Future*. https://doi.org/10.1201/9781003643173-3

Ahmad Laskar, A., Ahmed, M., Vo, D. V. N., Abdullah, A., Shahadat, M., Mahmoud, M. H., Khan, W., & Yusuf, M. (2023). Mathematical modeling and regression analysis using MATLAB for optimization of microwave drying efficiency of banana. *Thermal Science and Engineering Progress*, *46*. https://doi.org/10.1016/j.tsep.2023.102157

Alberto, S. P. C. (2015). "USO DE LA TECNOLOGÍA E INFORMÁTICA COMO FACTOR DETERMINANTE EN LA CALIDAD ESTUDIANTIL DEL NIVEL MEDIO EN EL INSTITUTO JACOBO ARBENZ GUZMÁN, QUETZALTENANGO". *World Agriculture*, *1*(May).

Al-Ezzi, A. S., & Ansari, M. N. M. (2022). Photovoltaic Solar Cells: A Review. In *Applied System Innovation* (Vol. 5, Issue 4). https://doi.org/10.3390/asi5040067

Almeida, C. S. de, Miccoli, L. S., Andhini, N. F., Aranha, S., Oliveira, L. C. de, Artigo, C. E., Em, A. A. R., Em, A. A. R., Bachman, L., Chick, K., & Curtis, D. (2018). Republic of Kenya, Ministry of Environment , Water and Natural Resources Department of Environment and Natural Resources, Policy Draft April 2017. *Universty of South Africa.*, *5*(4).

Alwi, Z. (2016). studi ilmu hadist. In *Revista Brasileira de Linguística Aplicada* (Vol. 5, Issue

1).

Amblard, F., & Quattrociocchi, W. (2018). Simulating social complexity: Social Networks and Spatial Distribution. In *Simulating Complexity*.

Analysis of international experience in implementing Artificial Intelligence in the educational process. (2025). *Scientia et Societus*. https://doi.org/10.69587/ss/1.2025.76

Andrade, F., Correa, A., García, N., Rodilla, V., Villagrasa, V., & Nuria Linares, G. (2020). Plantas Medicinales. In *Arts in Psychotherapy* (Vol. 291, Issue April).

Apaza, I. M. A., Choque, H. W. Z., & Laqui, A. J. C. (2025). Transforming university education: a systematic review of mathematical modeling in learning. *International Journal of Evaluation and Research in Education*, *14*(3). https://doi.org/10.11591/ijere.v14i3.32523

Apiola, M., López-Pernas, S., & Saqr, M. (2023). Past, present and future of computing education research: A global perspective. In *Past, Present and Future of Computing Education Research: A Global Perspective*. https://doi.org/10.1007/978-3-031-25336-2

Apriliyani. (2015). Sistem Penentuan Tingkat Resiko Penyakit Jantung Koroner Dan Kardiovaskuler Menggunakan Metode Framingham Score. *World Agriculture*, *1*(May).

Arayici, Y., Hamilton, A., & Gamito, P. (2006). Modelling 3D scanned data to visualise and analyse the Built Environment for regeneration. *Surveying and Built Environment*, *17*(2).

ARIAS ÁLVAREZ, M. C. (2010). SUPERHEATED STEAM GENERATION IN A FRESNEL SOLAR FIELD FOR HYBRIDIZATION OF A COAL POWER PLANT. *World Agriculture*, *1*(May).

Asamoah, G., Baah-Adade, A. K., & Ayikpah, Y. A. A. (2021). Effects of Emotional Intelligence Variables on Investor Behaviour in Ghana. In *Available Journal of Technology, Engineering, Management & Computational Sciences ISSN* (Vol. 1, Issue 1).

Aspling, F. (2020). Unleashing Animal-Computer Interaction: A Theoretical Investigation of the “I” in ACI. In *Report Series / Department of Computer & Systems Sciences* (Issue 20).

Baggio, R. (2018). Measuring tourism: Methods, indicators, and needs. In *The Future of Tourism: Innovation and Sustainability*. https://doi.org/10.1007/978-3-319-89941-1_13

Baggio, R. (2019). Measuring Tourism Methods, Indicators, and Needs: Innovation and Sustainability. In *The Future of Tourism*.

Baimakhanova, A. B., Kulton, Y., & Rakhimzanova, L. B. (2025). Training of future doctors in computer modeling. *R&E-SOURCE*. https://doi.org/10.53349/re-source.2025.is1.a1373

Barbosa, M. I. G., Senna, M. L. G. S. de, Dias, R. C., Cavalcante, R. P., & Castilho, W. S. (2025). MODELAGEM MATEMÁTICA E FORMAÇÃO OMNILATERAL – ESTUDO DE CASO SOBRE O PROCESSO DE ENSINO-APRENDIZAGEM DE MATEMÁTICA NO ENSINO MÉDIO INTEGRADO. *Revista Ibero-Americana de Humanidades, Ciências e Educação*, *11*(5). https://doi.org/10.51891/rease.v11i5.19118

Bardini, R., & di Carlo, S. (2024). Computational methods for biofabrication in tissue engineering and regenerative medicine - a literature review. In *Computational and Structural Biotechnology Journal* (Vol. 23). https://doi.org/10.1016/j.csbj.2023.12.035

Bastedo, M. N., & Bowman, N. A. (2015). Improving Admission of Low-SES Students at Selective Colleges: Results from a Natioanl Experiment. *Educational Researcher*, *33*.

Bilan, N., Volkova, N., Fedoreiko, I., Horbatiuk, R., Bureha, N., Zahorodnii, R., & Zamora, Y. (2025). Methodology of foreign language training of master’s students in the field of energy using innovative technologies. *Edelweiss Applied Science and Technology*, *9*(3). https://doi.org/10.55214/25768484.v9i3.5154

Biryukov, V. (2025). The importance of hierarchy and classification of sciences in the educational process. *International Science Journal of Education & Linguistics*, *4*(3). https://doi.org/10.46299/j.isjel.20250403.02

Bondada, B. R., Dwa, S., Magar, U. T., Denning, J., Tania, M., & Fails, J. A. (2025). Soul Support: Designing Hopeful Wearables for Children's Emotional Wellness. *Proceedings - 24th Annual ACM Interaction Design and Children Conference, IDC 2025*. https://doi.org/10.1145/3713043.3737389

BORGHI, C. M. S. D. O. (2015). DETERMINAÇÃO SOCIAL DO PROCESSO SAÚDE-DOENÇA: QUE VOZES ECOAM NO BRASIL? *World Agriculture*, *1*(May).

Brandes, U., & Erlebach, T. (2005). Network Analysis: Methodological Foundations. In *Lecture Notes in Computer Science* (Vol. 3418).

Bransford, J. D., Barron, B., Pea, R. D., Meltzoff, A., Kuhl, P., Bell, P., Stevens, R., Schwartz, D. L., Vye, N., Reeves, B., Roschelle, J., & Sabelli, N. H. (2006). Foundations and opportunities for an interdisciplinary science of learning. In *The Cambridge handbook of the learning sciences*.

Breckling, B., Jopp, F., & Reuter, H. (2011). Historical background of ecological modelling and its importance for modern ecology. In *Modelling Complex Ecological Dynamics: An Introduction into Ecological Modelling for Students, Teachers & Scientists*. https://doi.org/10.1007/978-3-642-05029-9_3

Brega, O. N., & Kruglyakova, G. v. (2025). Digital competence development while video podcast creation in ESP. *Perspektivy Nauki i Obrazovania*, *74*(2). https://doi.org/10.32744/pse.2025.2.14

Briones, M. (2015). Diseño e Implementación de un Plan Estratégico de Comunicación para Fortalecer los Mecanismos de Comunicación e Identidad de las Empresas Comercializadoras de Ropa Deportiva, caso Marathon Sports. *UTE*, *1*(May).

Brovko, K., Vynokurova, N., & Velykohorskyi, O. (2025). Computer simulation of the development of the Chornobyl nuclear power plant accident using an interactive simulator. *Bulletin of the National Technical University "KhPI". Series: Energy: Reliability and Energy Efficiency*, *1(10)*. https://doi.org/10.20998/eree.2025.1(10).329980

Bystrushkina, N., Vasilyeva, T., & Holina, S. (2025). The style of self-regulation of behavior among students of a medical university engaged in sports. *Scientific Notes of P. F. Lesgaft University*. https://doi.org/10.5930/1994-4683-2025-254-260

C. B., & N. J. S. (2019). Computer Simulation Validation Fundamental Concepts, Methodological Frameworks, and Philosophical Perspectives. In *Simulation Foundations, Methods and Applications*.

César Augusto, V. V. (2015). La Curva Hibrida De Riesgo: Análisis Retrospectivo Y Prospectivo Del Riesgo Por Fenómenos Naturales. *Tesis*, *1*(May).

Çevik, M., Savaşaneril, N. B., & Sezer, M. (2025). A Review of Polynomial Matrix Collocation Methods in Engineering and Scientific Applications. In *Archives of Computational Methods in Engineering* (Vol. 32, Issue 6). https://doi.org/10.1007/s11831-025-10235-6

Chen, S.-H., Terano, T., Yamamoto, R., & Tai, C.-C. (2014). Advances in Computational Social Science: The Fourth World Congress. In *Agent based social systems* (Issue 11).

Chigambayeva, D., & Soltan, G. (2025). CONSTRACTION OF DISTRIBUTION MODELS OF THE UNIVERSITY EDUCATIONAL WORK VOLUME. *Scientific Journal of Astana IT University*. https://doi.org/10.37943/21hivr6985

Chigambayeva, D., Goryakin, M., & Batova, P. (2025). EDUCATIONAL FLOWS DISTRIBUTION SYSTEM OF UNIVERSITY ACADEMIC GROUPS. *Scientific Journal of Astana IT University*. https://doi.org/10.37943/22vhyr8574

Chushchak, I., & Andrunyk, V. (2025). Modeling an information technology system to support the educational process for individuals with special needs. *Vìsnik Nacìonal'nogo Unìversitetu "L'vìvs'ka Polìtehnìka". Serìâ Ìnformacìjnì Sistemi Ta Merežì*, *17*. https://doi.org/10.23939/sisn2025.17.330

Connop, S., Vandergert, P., Eisenberg, B., Collier, M. J., Nash, C. C., Clough, J., Newport, D. J., Cameron, R. W. F., Blanuša, T., McClymont, K., Hansen, R., Pauleit, S., Marsboom, C., Vrebos, D., Staes, J., Meire, P., Wang, J., Banzhaf, E., Iojă, C. I., … McLean, N. (2017). Sustainability. *Sustainability*, *9*(1).

Courbet D. (2013). Pour un développement raisonné des méthodes expérimentales en SIC. Quelques intérêts épistémologiques du pluralisme méthodologique. *ESSACHESS - Journal for Communication Studies*, *6*(1). https://doi.org/10.2139/ssrn.2314403

Dahlberg, B., Wittink, M. N., & Gallo, J. J. (2015). Funding and Publishing Integrated Studies: Writing Effective Mixed Methods Manuscripts and Grant Proposals. In *SAGE Handbook of Mixed Methods in Social & Behavioral Research*. https://doi.org/10.4135/9781506335193.n30

Darusman, B. (2015). Persepsi Mahasiswa Terhadap Kontribusi Hmp Pgsd Dalam Mengoptimalkan Kemampuan Berpikir Kritis Mahasiswa Pgsd Fkip Universitas Muhammadiyah Surakarta. In *Universitas Muhammadiyah Surakarta* (Vol. 1, Issue May).

Davis, G. B. (1985). Management Information System: Conceptual Foundation, Structure, and Development (Second). In *Mc. Graw-Hill, Inc*.

de Luca, L. (2023). A digital ecosystem for the interdisciplinary study of Notre-Dame de

Paris | Un ecosistema digitale per lo studio interdisciplinare di Notre-Dame de Paris. In *Disegnare Idee Immagini* (Vol. 34, Issue 66).

de Roo Gert, Ward. S. Rauw. (2012). Springer Complexity program. In *Media* (Issue September 2007).

Despoina G. Mavridou. (2025). The necessity of modeling at school and the teacher's role. *International Journal of Science and Research Archive*, *16*(2). https://doi.org/10.30574/ijsra.2025.16.2.2326

Dond, D. K., Gulhane, N. P., & Dhamejani, C. (2018). Mathematical Modelling & MATLAB Simulation of Diesel Engine. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3101296

Duad, A. M. (2015). Karakteristik Fisisk Daging Sapi Bali Pascarigor Yang Dimarinasi Theobromin Pada Level Dan Lama Marinasi Yang Berbeda. *World Agriculture*, *1*(May).

Edmonds, B., & Meyer, R. (2017). Simulating social complexity - A Handbook. In *Simulating Complexity*.

Emberley, A. C., & Moore, T. J. (2023). Professor Talk in Undergraduate, Introductory Design: A Multiple Case Study from Mechanical and Biomedical Engineering. *Studies in Engineering Education*, *4*(2). https://doi.org/10.21061/see.74

Fadhiliani, D., Umam, K., Suhartati, S., & Johar, R. (2025). Mathematical Modeling of Student Learning Outcomes using E-learning-Based Remedial Programs. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, *9*(3). https://doi.org/10.31764/jtam.v9i3.31404

Faroughi, S. A., Pawar, N. M., Fernandes, C., Raissi, M., Das, S., Kalantari, N. K., & Mahjour, S. K. (2024). Physics-Guided, Physics- Informed, and Physics-Encoded Neural Networks and Operators in Scientific Computing: Fluid and Solid Mechanics. *Journal of Computing and Information Science in Engineering*, *24*(4). https://doi.org/10.1115/1.4064449

Frankhauser, P., Tannier, C., Vuidel, G., Houot, H., Kropf, K., Maretto, M., Strappa, G., Whitehand, J., Springer, & Tannier, C. (2018). About fractal models in urban geography and planning : refuting the aesthetics and the universal norm. In *Computers, Environment and Urban Systems* (Vol. 67, Issue 2017).

Friberg, E. E., & Saewert, K. J. (2023). Conceptual Foundations. In *Conceptual Foundations: The Bridge to Professional Nursing Practice*. https://doi.org/10.7551/mitpress/11171.003.0005

Garzón, Y. (2015). ARTETERAPIA COGNITIVA CONDUCTUAL PARA EL TRATAMIENTO DE ANSIEDAD Y DEPRESIÓN EN UN GRUPO DE PACIENTES ONCOLÓGICOS DE LA ASOCIACIÓN NACIONAL CONTRA EL CÁNCER CAPÍTULO DE VERAGUAS. *World Agriculture*, *1*(May).

Gasperoni, F., Ieva, F., & Maria Paganoni, A. (2025). Exercise Book of Statistical Inference. In *UNITEXT - La Matematica per il 3 piu 2* (Vol. 173). https://doi.org/10.1007/978-3-031-86670-8

Gaudette, G., & Kafel, D. (2012). A Case Study: Data Management in Biomedical Engineering. *Journal of eScience Librarianship*, *1*(3). https://doi.org/10.7191/jeslib.2012.1027
Geographic Citizen Science Design. (2021). In *Geographic Citizen Science Design*. https://doi.org/10.2307/j.ctv15d8174

Gerdsri, N., Kongthon, A., & Puengrusme, S. (2017). Profiling the Research Landscape in Emerging Areas Using Bibliometrics and Text Mining: A Case Study of Biomedical Engineering (BME) in Thailand. *International Journal of Innovation and Technology Management*, *14*(2). https://doi.org/10.1142/S0219877017400119

Giacomarra, E. J. (2015). Heritage Education for School-Aged Children: an Analysis of Program in Salem, Massachusetts. *World Agriculture*, *1*(May).

Gokulakrishnan. J., & C. Gantham. P. (2024). Ergonomics Evaluation an Origin and Overview - Copy. *International Journal For Multidisciplinary Research*, *6*(2). https://doi.org/10.36948/ijfmr.2024.v06i02.15960

Gomes, Í. de A. (2015). Desenvolvimento Experimental de um Secador Solar de Frutas com Aproveitamento Multienergético. In *Climate Change 2013 - The Physical Science Basis* (Vol. 1, Issue May).

Goncalves Dos Santos, K. C. (2015). Interaction of Ilex paraguariensis with beneficial microorganisms. *Nhk 技研*, *151*(1).

Gorard, S., Siddiqui, N., & See, B. H. (2015). Education Endowment Foundation Durham Shared Maths Project Evaluation report and Executive summary. *Review of Educational Research*, *1*(2).

Gordon, J. S. (2020). Building Moral Robots: Ethical Pitfalls and Challenges. *Science and Engineering Ethics*, *26*(1). https://doi.org/10.1007/s11948-019-00084-5

Graha, arya bintang, Ginting, M., & Tobing, E. (2015). ANALISA PRESSURE BUILD UP DAN INTERFERENCE TEST PADA SUMUR ALPHA DAN "BETA LAPANGAN X. *Seminar Nasional Cendekiawan*, *1*(May). https://doi.org/10.1017/CBO9781107415324.004

Grams, D., Farrell, B., Mackintosh, J. D., Berry, C., Liscutin, N., Aydinlioğlu, Ö., Broadhead, S., Puccio, G., Ravnborg, J. L., Monaghan, S., Bursk, E. C., Sethi, B. S., Dunbar, D. S., Bergh, B. G. v, Reid, L. N., Schorin, G. A., Roberts, J., Roberts, T., Bergh, B. G. v, … Meron, Y. (2024). Scopus Document. *Journal of Advertising*, *12*(1).

Gratch, J., & Marsella, S. (2014). Social Emotions in Nature and Artifact. In *Social Emotions in Nature and Artifact*. https://doi.org/10.1093/acprof:oso/9780195387643.001.0001

Greco, C. (2024). Politics and practices of the ethnographies of biomedicine and STEM: Among white coats. In *Politics and Practices of the Ethnographies of Biomedicine and STEM: Among White Coats*. https://doi.org/10.1007/978-3-031-65797-9

Gross, T., & Sayama, H. (2009). Adaptive networks: Theory, models, and data. In *Springer Complexity Series*.

Haken, H. (2020). (Springer series in synergetics, Springer complexity) T.D. Frank - Nonlinear Fokker-Planck equations_ fundamentals and applications-Springer (2005).pdf. In *Media*.

Hardcastle, V. G. (2006). The theoretical and methodological foundations of cognitive neuroscience. In *Philosophy of Psychology and Cognitive Science*. https://doi.org/10.1016/B978-044451540-7/50027-X

Hemsath, T. L. (2012). Using prototyping to teach digital fabrication techniques. *Electronic Journal of Information Technology in Construction*, *17*.

Historical and Philosophical Dimensions of Logic, Methodology and Philosophy of Science. (1977). In *Historical and Philosophical Dimensions of Logic, Methodology and Philosophy of Science*. https://doi.org/10.1007/978-94-017-1780-9

Holeman, I. (2018). Room for Silence: Ebola Research, Pluralism and the Pragmatic Study of Sociomaterial Practices. *Computer Supported Cooperative Work: CSCW: An International Journal*, *27*(3–6). https://doi.org/10.1007/s10606-018-9329-x

Hopkins, K., Mutasam, U., Ibrohim, I., Susilo, H., Puttick, S., Cullinane, A., Voss, S., Kruse, J. W., Kent-Schneider, I., Wei, B., Jiang, Z., Gai, L., Torres, J., Vasconcelos, C., Cavallo, A. A., Burgers, H., Ghezzi, A., van de Vrande, V., Erduran, S., … Schmidt Kjærgaard, R. (2020). Exploring the Use of Lesson Study to Develop Elementary Preservice Teachers' Pedagogical Content Knowledge for Teaching Nature of Science. *Science and Education*, *16*(2).

Hoyte, S. (2021). Co-designing extreme citizen science projects in Cameroon: biodiversity conservation led by local values and indigenous knowledge. In *Geographic Citizen Science Design*. https://doi.org/10.2307/j.ctv15d8174.8

Iacovidou, E., Millward-Hopkins, J., Busch, J., Purnell, P., Velis, C. A., Hahladakis, J. N., Zwirner, O., & Brown, A. (2017). A pathway to circular economy: Developing a conceptual framework for complex value assessment of resources recovered from waste. *Journal of Cleaner Production*, *168*. https://doi.org/10.1016/j.jclepro.2017.09.002

Idoko Peter Idoko, Gerald Chekwube Ezeamii, Christian Idogho, Enemali Peter, Ubong Sunday Obot, & Vitalis Afebuame Iguoba. (2024). Mathematical modeling and simulations using software like MATLAB, COMSOL and Python. *Magna Scientia Advanced Research and Reviews*, *12*(2). https://doi.org/10.30574/msarr.2024.12.2.0181

Ilkiv, O., Matviyiv, V., & Yasinskyi, A. (2025). «Computer technology and mathematical statistics» as a means of forming innovative competence in higher education institutions of physical culture and sports. *Scientific Journal of National Pedagogical Dragomanov University. Series 15. Scientific and Pedagogical Problems of Physical Culture (Physical Culture and Sports)*, *5K(191)*. https://doi.org/10.31392/udu-nc.series15.2025.05k(191).18

Interculturales, C. de E. M. (2020). Manual para la promoción del buen cultivo y uso de plantas medicinales. In *Arts in Psychotherapy*.

Jadhav, D., & Thakare, M. (2022). Mathematical Modeling and Simulation of a Fuel Cell in MATLAB. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4056056

Jakub Kolacz. (2015). ENERGY MINIMIZATION IN NEMATIC LIQUID CRYSTAL SYSTEMS DRIVEN BY GEOMETRIC CONFINEMENT AND TEMPERATURE GRADIENTS WITH APPLICATIONS IN COLLOIDAL SYSTEMS. *World Agriculture*, *1*(May).

Jarke, J., Prietl, B., Egbert, S., Boeva, Y., Heuer, H., & Arnold, M. (2025a). Algorithmic

Regimes: Methods, Interactions, and Politics. In *Algorithmic Regimes: Methods, Interactions, and Politics*. https://doi.org/10.4324/9781003690672

Jarke, J., Prietl, B., Egbert, S., Boeva, Y., Heuer, H., & Arnold, M. (2025b). Algorithmic Regimes: Methods, Interactions, and Politics. In *Algorithmic Regimes: Methods, Interactions, and Politics*. https://doi.org/10.4324/9781003690672

Jung, E. E. J., Jung, E. E. J., Ye, J., Kurashima, Y., Kobayashi, T., Tsuda, H., Takahara, T., Sakurai, W., Lima, L. M. M. de, Sá, L. R. de, Macambira, A. F. U. dos S., Nogueira, J. de A., Vianna, R. P. de T., Moraes, R. M. de, Barzegar, M., Sadeghi-Niaraki, A., Shakeri, M., Choi, S.-M., Oduntan, G., … Buchanan, R. (2020). Abstracts. *Journal of Planning Literature*, *10*(1).

Kalinin, S. I., & Toropova, S. I. (2025). Integration of computational thinking into the mathematical education of students of natural sciences. *Perspektivy Nauki i Obrazovania*, *74*(2). https://doi.org/10.32744/pse.2025.2.11

Kaminski, M. R., Boscarioli, C., & Klüber, T. E. (2025). Computational Thinking in Mathematical Modeling Practices from the Perspective of Meaningful Learning Theory. *Revista Internacional de Pesquisa Em Educação Matemática*, *15*(3). https://doi.org/10.37001/ripem.v15i3.4452

Kaplan, B. (2001). Evaluating informatics applications - Some alternative approaches: Theory, social interactionism, and call for methodological pluralism. *International Journal of Medical Informatics*, *64*(1). https://doi.org/10.1016/S1386-5056(01)00184-8

Kasza, G. (2001). Perestroika: For An Ecumenical Science of Politics. *PS: Political Science and Politics*, *34*(3).

Khairulin, O. (2023). Ontological modeling of the psychological field of a play. *Psihologìâ ì Suspìl'stvo*, *2*(2023). https://doi.org/10.35774/pis2023.02.106

Klochko, V. I., Klochko, O. v., & Fedorets, V. M. (2025). Mathematics for digital signal processing: series and integrals of Fourier in professional training. *Journal of Physics: Conference Series*, *3105*(1). https://doi.org/10.1088/1742-6596/3105/1/012007

Koumpis, A. (2012). Management Information Systems for Enterprise Applications: Business Issues, Research and Solutions. In *Management Information Systems for Enterprise Applications: Business Issues, Research and Solutions*. https://doi.org/10.4018/978-1-4666-0164-2

Kovtoniuk, M. M., Soia, O. M., Kosovets, O. P., & Tiutiun, L. A. (2025). Virtual educational mathematics park as a platform for educational and research activities at the university. *Journal of Physics: Conference Series*, *3105*(1). https://doi.org/10.1088/1742-6596/3105/1/012024

Kroemer, K. H. E. (1989). A Survey of Ergonomic Models of Anthropometry, Human Biomechanics, and Operator-Equipment Interfaces. In *Applications of Human Performance Models to System Design*. https://doi.org/10.1007/978-1-4757-9244-7_23

Kulas, J., Fetzer, J. H., & Rankin, T. L. (2011). Philosophy, Language, and Artificial Intelligence: Resources for Processing Natural Language. In *Studies in Cognitive Systems*.

Kurasov, D. (2020). Mathematical modeling system MatLab. *Journal of Physics: Conference Series*, *1691*(1). https://doi.org/10.1088/1742-6596/1691/1/012123

Kuroda, T. (2024). Slashing IC Power and Democratizing IC Access for the Digital Age. *IPSJ Transactions on System LSI Design Methodology*, *17*. https://doi.org/10.2197/ipsjtsldm.17.2

Laja, Y. P. W., Delvion, E. B. S., & Siahaan, M. M. L. (2025). Chatbot Artificial Intelligence (AI) to Supporting Differentiated Mathematics Learning. *Jurnal Pendidikan MIPA*, *26*(1).

https://doi.org/10.23960/jpmipa.v26i1.pp118-130

Lamas, D., & Tomberg, V. (2024). HCI Design Perspectives, Trends and Approaches. In *Foundations and Fundamentals in Human-Computer Interaction*. https://doi.org/10.1201/9781003495109-11

Levi-Faur, D. (2006). A question of size? A heuristics for stepwise comparative research design. In *Innovative Comparative Methods for Policy Analysis: Beyond the Quantitative-Qualitative Divide*. https://doi.org/10.1007/0-387-28829-5_3

Lin, D. (2024). Application of Mathematical Modeling Based on MATLAB in Experimental Data Analysis. *Procedia Computer Science*, *247*(C). https://doi.org/10.1016/j.procs.2024.10.011

Linares, N. (2013). Plantas Medicinales.Farmacia de la naturaleza. *UPA MADRID*, *291*(April).

Linares, N. (2020). Plantas Medicinales, Cuaderno de trabajo. *Arts in Psychotherapy*, *291*(April).

Lubis, M. Z. (2015). Tingkat Kesukaan dan Daya Terima Makanan serta hubungannya ddengan Kecukupan Energi dan Zat Gizi pada Santri Putri MTs Darul Muttaqien Bogor. *World Agriculture*, *1*(May).

Lund, S. (2015). THE COMEDY OF FRENCHNESS: EXPLORING SOCIAL ISSUES IN MAINSTREAM FRENCH CINEMA. *World Agriculture*, *1*(May).

Ma, H., Li, L., Ma, Y., Liu, X., & Yang, S. (2025). Exploration and Application of Project-Based Teaching Integrating Scientific Research Achievements in Electronic Comprehensive Design Course. *Computer Applications in Engineering Education*, *33*(3). https://doi.org/10.1002/cae.70047

MANUEL, V. M. J. (2015). HERRAMIENTAS FINANCIERAS PARA LA COMPAÑÍA "AMARY´S", DE LA CIUDAD DE QUITO PROVINCIA DE PICHINCHA. *UNIVERSIDAD REGIONAL AUTÓNOMA DE LOS ANDES "UNIANDES - IBARRA," 1*(May).

Marquis, P., Papini, O., & Prade, H. (2020). A Guided Tour of Artificial Intelligence Research: Volume III: Interfaces and Applications of Artificial Intelligence. In *A Guided Tour of Artificial Intelligence Research: Volume III: Interfaces and Applications of Artificial Intelligence* (Vol. 3). https://doi.org/10.1007/978-3-030-06170-8

Martins Bessa, S., Soares Oliveira, F. W., & Floriano Paulino, O. (2025). Mathematical modeling: a study on its incorporation into basic education from the proceedings of SIPEM. *Educação Matemática Pesquisa*, *27*(1).

Martins, A. C. R. (2025). Human flaws and logical tools in engineering. *Canadian Journal of Chemical Engineering*, *103*(8). https://doi.org/10.1002/cjce.25764

Mathematical Methods in Signal Processing and Digital Image Analysis. (2008). In *Mathematical Methods in Signal Processing and Digital Image Analysis*. https://doi.org/10.1007/978-3-540-75632-3

Matorin, D. D., & Cherepkov, A. Yu. (2025). Numerical analysis of multicomponent dynamic models of stage-by-stage knowledge assimilation. *Nonlinear World*. https://doi.org/10.18127/j20700970-202501-04

McCloskey, D. (1999). Other Things Equal - Economical Writing: An Executive Summary. *Eastern Economic Journal*, *25*(2).

Montesinos, L., Santos-Diaz, A., Salinas-Navarro, D. E., & Cendejas-Zaragoza, L. (2022). Experiential Learning in Biomedical Engineering Education Using Wearable Devices: A

Case Study in a Biomedical Signals and Systems Analysis Course. *Education Sciences*, *12*(9). https://doi.org/10.3390/educsci12090598

Moralioglu, B., & Gül, L. F. (2026). Envisioning tomorrow's spaces: A design fiction framework for exploring the integration of spatial computing into future architecture. *Futures*, *175*. https://doi.org/10.1016/j.futures.2025.103723

Moss, L. S., & Johnson, D. E. (1995). Evolving Algebras and Mathematical Models of Language. In *Applied Logic: How, What and Why*. https://doi.org/10.1007/978-94-015-8533-0_7

Mu'arif, W., Mustaqimah, Prasetyawati, A., Suryani, Maryati, T., & Rokhimah, S. (2025). Penerapan Model Realistic Mathematic Education (RME) untuk Meningkatkan Hasil Belajar pada Pembelajaran Matematika. *Jurnal Educazione : Jurnal Pendidikan, Pembelajaran Dan Bimbingan Dan Konseling*, *2*(2).

Mutanov, G. (2015). Mathematical Methods and Models in Economic Planning, Management and Budgeting. In *Mathematical Methods and Models in Economic Planning, Management and Budgeting*. https://doi.org/10.1007/978-3-662-45142-7

Nantwi, S. K. N. and W. K. (2013). British Journal of Education, Learning and Development Psychology. *Jurnal EMBA*, *1*(4).

Nevid, J. S., Pavlov, I., Powerlecture, F., Desktop, F., Weber, M. S., Jin, Z., Psy, S. I., Psy, F., Psychology, C., Fa, P. S. Y., Fa, P. S. Y., Reports, A., Action, T., Report, T. B., Stowell, J. R., Kim, O., Kendeou, P., van den Broek, P. P. D. M., White, M. J., … McNamara, D. S. (2011). An introduction to the psych package: Part II. Scale construction and psychometrics. *Science Education*, *1*(1). https://doi.org/10.1017/CBO9781107415324.004

Nguyen, X. H., & Nguyen, M. P. (2015). Mathematical modeling of photovoltaic cell/module/arrays with tags in Matlab/Simulink. *Environmental Systems Research*, *4*(1).

https://doi.org/10.1186/s40068-015-0047-9

Nilsson, F., & Dunin-Woyseth, H. (2012). Building a culture of doctoral scholarship in architecture and design. A Belgian-Scandinavian case. *Nordic Journal of Architectural Research*, *23*(1).

Nisa, A. N. (2015). ANALISIS USER EXPERIENCEMEDIA SOSIAL MINDTALK. *World Agriculture*, *1*(May).

Núñez, G. (2015). Dimensiones de personalidad y estilos de afrontamiento en pacientes con diagnóstico de cáncer. *World Agriculture*, *1*(May).

Nyerges, T. L., Couclelis, H., & McMaster, R. (2011). The SAGE handbook of GIS and society. In *The SAGE Handbook of GIS and Society*. https://doi.org/10.4135/9781446201046

Nyirenda, J. L. Z., Mbakaya, B. C., Wagner, D., Stete, K., Jaeger, V. K., Bartz, A., Karch, A., Nanyinza, E., Soko, M., Chimbatata, N., Chirambo, G. B., & Lange, B. (2025). NOZGEKA—a collaborative curriculum development process for a Master of Science in Public health with a focus on infectious diseases epidemiology in Malawi: a perspective. *Humanities and Social Sciences Communications*, *12*(1). https://doi.org/10.1057/s41599-025-05654-6

Ocaña, A. C., Mengual-Andres, S., & Segarra, J. R. (2024). Educational innovation to address complex societal challenges. In *Educational Innovation to Address Complex Societal Challenges*. https://doi.org/10.4018/9798369330739

Opderbeck, D. W. (2023). The End of the Law? Law, Theology, and Neuroscience. *Perspectives on Science and Christian Faith*, *75*(1). https://doi.org/10.56315/pscf3-23opderbeck

Orazali, G., Dauletkulova, A., & Mekebayev, N. (2025). METHODOLOGY OF

TEACHING GEOMETRY IN THE CONTEXT OF DIGITALIZATION OF EDUCATION. *Журнал Серии «Педагогические Науки»*, *76*(1). https://doi.org/10.48371/peds.2025.76.1.029

Orlova, E. v. (2022). Design Technology and AI-Based Decision Making Model for Digital Twin Engineering. *Future Internet*, *14*(9). https://doi.org/10.3390/fi14090248

Oshima, J., van Aalst, J., Mu, J., & Chan, C. K. K. (2023). Development of the learning sciences: Theories, pedagogies, and technologies. In *International Handbook on Education Development in the Asia-Pacific*. https://doi.org/10.1007/978-981-19-6887-7_49

Paraponaris, C., Ermine, J.-L., Guittard, C., & Lièvre, P. (2012). Knowledge management in a French research community: a case study of GeCSO congress: Very Informal Newsletter on Library Automation. *VINE*, *42*(3/4).

Pardo, L. (2015). ELABORACIÓN DEL MAPA TENENCIAL Y PRELIMINAR CATASTRAL DE LA PROVINCIA DE CHIRIQUI PARA EL PROYECTO DE REGULARIZACION DE TIERRAS DE LA REPÚBLICA DE PANAMÁ. *World Agriculture*, *1*(May).

Partos, R. (2019). Time to legislate. In *The Making of the Conservative Party's Immigration Policy*. https://doi.org/10.4324/9781351010658-3

Pasupuleti, M. K. (2024). Revolutionizing Industries with Digital Twin Technology. In *Digital Twin Technology*. https://doi.org/10.62311/nesx/97806

Paternò, L., Dhokia, V., Menciassi, A., Bilzon, J., & Seminati, E. (2023). Correction: A personalised prosthetic liner with embedded sensor technology: a case study (BioMedical Engineering OnLine, (2020), 19, 1, (71), 10.1186/s12938-020-00814-y). In *BioMedical Engineering Online* (Vol. 22, Issue 1). https://doi.org/10.1186/s12938-023-01154-3

Patriarca, R., di Gravio, G., Woltjer, R., Costantino, F., Praetorius, G., Ferreira, P., & Hollnagel, E. (2020). Framing the FRAM: A literature review on the functional resonance analysis method. *Safety Science*, *129*. https://doi.org/10.1016/j.ssci.2020.104827

Payette, M., & Abdul-Nour, G. (2023). Asset Management, Reliability and Prognostics Modeling Techniques. *Sustainability (Switzerland)*, *15*(9). https://doi.org/10.3390/su15097493

Pérez-Mojica, R., & Osorio-Gutiérrez, P. A. (2025). Diseño de actividades de modelación matemática para solucionar problemas reales de la función cuadrática. *Praxis*, *21*(2). https://doi.org/10.21676/23897856.6079

Peters, T. (2019). Data-driven science and engineering: machine learning, dynamical systems, and control. *Contemporary Physics*, *60*(4). https://doi.org/10.1080/00107514.2019.1665103

Petrucco, C., & Agostini, D. (2019). Augmented Reality Learning: Pedagogical Aspects and Technologies for a Future Methodological Framework. In *Virtual Worlds: The Virtual Reality and Augmented Reality Intersections*. https://doi.org/10.1163/9781848883840_007

Picciotto, R. (2018). Evaluation for an Open Society: Then and Now. *Journal of MultiDisciplinary Evaluation*, *14*(31). https://doi.org/10.56645/jmde.v14i31.503

Podkhodova, N., Sheremetyeva, O., & Soldaeva, M. (2025). How to teach mathematics so that students truly understand it. *South Florida Journal of Development*, *6*(9). https://doi.org/10.46932/sfjdv6n9-001

Pop, T., Buzduga, C., Pentiuc, R.-D., Ifrim, V.-C., & Ungureanu, C. (2022). Mathematical modeling in MATLAB of a photovoltaic panel. *IOP Conference Series: Materials Science and Engineering*, *1256*(1). https://doi.org/10.1088/1757-899x/1256/1/012037

Portugali, J. (2012). Complexity Theories of Cities: Implications to Urban Planning. In *Complexity Theories of Cities Have Come of Age*. https://doi.org/10.1007/978-3-642-24544-2_13

Prem, E. (2015). The role of technology for innovation management research methodology. *ISPIM Conference Proceedings*, *June*.

Primiero, G. (2019). On the Foundations of Computing. In *On the Foundations of Computing*. https://doi.org/10.1093/oso/9780198835646.001.0001

Pumain, D. (2012). Urban Systems Dynamics, Urban Growth and Scaling Laws: The Question of Ergodicity. In *Complexity Theories of Cities Have Come of Age*. https://doi.org/10.1007/978-3-642-24544-2_6

Pyka, A., & Scharnhorst, A. (2009). Innovation Networks; New Approaches in Modelling and Analyzing. In *Springer*.

Ringquist, E. J., & Anderson, M. R. (2013). Introduction. Meta-analysis for Public Management and Policy. In *Meta-Analysis for Public Management and Policy.*

Rosenberg, M. (2016a). Health geography II. *Progress in Human Geography*, *40*(4). https://doi.org/10.1177/0309132515581094

Rosenberg, M. (2016b). Health geography II: 'Dividing' health geography. *Progress in Human Geography*, *40*(4). https://doi.org/10.1177/0309132515581094

Rossy, Q., & Morselli, C. (2017). The contribution of forensic science to the analysis of crime networks. In *The Routledge International Handbook of Forensic Intelligence and Criminology*. https://doi.org/10.4324/9781315541945

Rubik, Programa Construye2025, Ertuğ KIYASOĞLU, UNEP, ONU Medio Ambiente, Minson, A. J., Latinoamericano, F., European Commission, E., Normativo, T., Nacional, C., Priess, J. A., Hauck, J., Haines-Young, R., Alkemade, R., Mandryk, M., Veerkamp, C., Gyorgyi, B., Dunford, R., Berry, P., … Europeiska Kommissionen. (2019). Hoja de Ruta RCD: Consulta Pública - Gestión Sustentable de los Recursos y Residuos, RCD, para una Economía Circular en Construcción. In *European Comission* (Vol. 53, Issue 1).

Saba, T., Saad, K. S. K., & Rashid, A. bin. (2024). Precise surface engineering: Leveraging chemical vapor deposition for enhanced biocompatibility and durability in biomedical implants. In *Heliyon* (Vol. 10, Issue 18). https://doi.org/10.1016/j.heliyon.2024.e37976

Saeki, A., Kawakami, T., & Lamb, J. (2025). The role of children's analogies in the transfer of their knowledge in mathematical modelling processes. *ZDM - Mathematics Education*, *57*(2). https://doi.org/10.1007/s11858-025-01675-2

Sahani, S. K., & Karna, S. K. (2025). Program for Cultural Education Optimization by the Application of Numerical Methods. *Journal of Posthumanism*, *5*(1). https://doi.org/10.63332/joph.v5i1.2687

Sakibayeva, B., & Sakibayev, S. (2025). Enhancing professional competence in physics and mathematics education through information technology. *Educational Research and Evaluation*, *30*(7–8). https://doi.org/10.1080/13803611.2025.2502813

Sardor, S., & Uldona, S. (2025). THE IMPORTANCE OF STUDYING MATHEMATICS AND MODERN APPROACHES. *JournalNX- A Multidisciplinary Peer Reviewed Journal*, *11*.

Shcherbatykh, S. v., Lykova, K. G., Igonina, E. v., & Simonovskaya, G. A. (2025). Development of cognitive mobility when teaching probability theory and mathematical statistics to students using digital solutions. *Perspektivy Nauki i Obrazovania*, *74*(2). https://doi.org/10.32744/pse.2025.2.12

Silander, M., Grindal, T., Gerard, S. N., & Salone, T. (2025). Learning Science and

Engineering From Videos and Games: A Randomized Trial of PBS KIDS The Cat in the Hat Knows a Lot About That! *Educational Researcher*, *54*(6). https://doi.org/10.3102/0013189X251327187

Singh, B., & Nayyar, A. (2024). Exploring diverse use cases of digital twins projecting digital transformation: Unlocking potential, addressing challenges and viable solutions. In *Digital Twins for Smart Cities and Villages*. https://doi.org/10.1016/B978-0-443-28884-5.00027-0

Smirnov, E. I., Skornyakova, A. Y. U., & Tikhomirov, S. A. (2025). The ontology and engineering of professional self-determination of school students in synergy manifestation of natural science with mathematical and digital methods. *Perspektivy Nauki i Obrazovania*, *3*. https://doi.org/10.32744/pse.2025.3.36

Sobran, N. M. M., & Ismail, Z. H. (2023). A Systematic Literature Review of Unsupervised Fault Detection Approach for Complex Engineering System. *Journal of Advanced Research in Applied Mechanics*, *103*(1). https://doi.org/10.37934/aram.103.1.4360

Solod, L. V., Tkachova, V. V., Prokofieva, H. Ya., & Bereziuk, H. H. (2025). MODELING THE STUDY OF SOLID MATERIALS' THERMAL CONDUCTIVITY USING THE DYNAMIC CALORIMETRY METHOD. *Ukrainian Journal of Civil Engineering and Architecture*, *3 (027)*. https://doi.org/10.30838/ujcea.2312.270425.129.1169

Stangl, F. J., & Riedl, R. (2025). *Hormone Measurements in the Research Field of Interruption Science: Review of Applied Methods*. https://doi.org/10.1007/978-3-031-71385-9_8

Stobbe, G. (2013). Just Enough ENGLISH GRAMMAR. In *Government Information Quarterly* (Vol. 4, Issue 1).

Sugiarti. (2015). Hubungan antara Kondisi Perairan dan Hasil Tangkapan Ikan di Muara - Muara Sungai Di Teluk Banten. *Institut Pertanian Bogor*.

Sumaryani, S., Manangin, yunita restiasa, & Astuti, S. (2015). senam nifas pada ibu post partum di desa meijing,yogyakarta. In *World Agriculture* (Vol. 1, Issue May).

Swanson, T. (2015). Consensus-as-a-service: a brief report on the emergence of permissioned, distributed ledger systems. Work. *World Agriculture*, *1*(May).

Syifa Afidah Nurul Arifin. (2025). Systematic Literature Review: Kemampuan Pemodelan Matematika Siswa SMP. *Griya Journal of Mathematics Education and Application*, *5*(3). https://doi.org/10.29303/griya.v5i3.802

Tashakkori, A., & Teddlie, C. (2015). Epilogue: Current Developments and Emerging Trends in Integrated Research Methodology. In *SAGE Handbook of Mixed Methods in Social & Behavioral Research*. https://doi.org/10.4135/9781506335193.n31

Taylor, J. R., Milne, A. J., & Macritchie, J. (2023). New musical interfaces for older adults in residential care: assessing a user-centred design approach. *Disability and Rehabilitation: Assistive Technology*, *18*(5). https://doi.org/10.1080/17483107.2021.1881172

Testov, V. A., & Perminov, E. A. (2023). TRANSDISCIPLINARY ROLE OF PHYSICAL AND MATHEMATICAL DISCIPLINES IN MODERN NATURAL SCIENCE AND ENGINEERING EDUCATION. *Obrazovanie i Nauka*, *25*(7). https://doi.org/10.17853/1994-5639-2023-7-14-43

Tian, Y., Chao, M. A., Kulkarni, C., Goebel, K., & Fink, O. (2022). Real-time model calibration with deep reinforcement learning. *Mechanical Systems and Signal Processing*, *165*. https://doi.org/10.1016/j.ymssp.2021.108284

Tiara, R. (2015). Pengaruh Relationship Marketing Terhadap Customer Retention (Retensi Pelanggan) Pada Wardah Beauty House Yogyakarta. In *World Agriculture* (Vol. 1, Issue May).

Tkaczyk, V. (2018). Whose larynx is it? Fields of scholarly competence around 1900. *History of Humanities*, *3*(1). https://doi.org/10.1086/696302

Turobov, A., Chumakova, M., & Vecherin, A. (2019a). World best practices in applying mathematical and statistical crime prediction algorithms. *Mezhdunarodnye Protsessy*, *17*(4). https://doi.org/10.17994/IT.2019.17.4.59.9

Turobov, A., Chumakova, M., & Vecherin, A. (2019b). World best practices in applying mathematical and statistical crime prediction algorithms; [МЕЖДУНАРОДНЫЙ ОПЫТ ПРИМЕНЕНИЯ МАТЕМАТИКОСТАТИСТИЧЕСКИХ АЛГОРИТМОВ ПРОГНОЗИРОВАНИЯ ПРЕСТУПНОСТИ]. *Mezhdunarodnye Protsessy*, *17*(4).

Ulker, D., & Ozaltun Celik, A. (2025). A qualitative metasynthesis: Interaction between mathematical modeling and higher order thinking. *Thinking Skills and Creativity*, *56*. https://doi.org/10.1016/j.tsc.2025.101773

Univ Lampung. (2015). Metode Pengukuran Rendemen Rumput Laut. *Skripsi*, *1*(May).

Valkova-Jarvis, Z., Nenova, M., & Mihaylova, D. (2025). Hypercomplex Numbers—A Tool for Enhanced Efficiency and Intelligence in Digital Signal Processing. *Mathematics*, *13*(3). https://doi.org/10.3390/math13030504

Varenne, F. (2003b). La simulation informatique face à la « méthode des modèles ». Le cas de la croissance des plantesThe computer simulation facing the “Method of Models”. The case of plant growth. *Nature Sciences Sociétés*, *11*(1). https://doi.org/10.1016/s1240-1307(03)00004-9

VELOSO, F., BARBOSA FILHO, F. D. H., & PERUCHETTI, P. (2022). Impactos da educação no mercado de trabalho. *Instituto Brasileiro de Economia*.

Videla-Reyes, R. (2023). CPA Methodology: educational technological design proposal to solve problems. *Pacific Journal of Technology Enhanced Learning*, *5*(1). https://doi.org/10.24135/pjtel.v5i1.169

Villarreal, M. E., & Coirini Carreras, A. (2025). Mathematical modeling in education: contributions and visibility of an Argentine research group. *ZDM - Mathematics Education*, *57*(7). https://doi.org/10.1007/s11858-025-01733-9

VILLENA, C. (2015). DISEÑO DE UN PLAN DE MARKETING ESTRATÉGICO PARA LA ESTACIÓN DE SERVICIOS VIGUESAM. *World Agriculture*, *1*(May).

Villota-Cadena, Á. P., Sandoval-Palis, I. P., Grijalva-Villegas, G. F., & Herrera-Granda, E. P. (2025). CDR-Solv: Solving the Convection-Diffusion-Reaction Equation with Algebraic Sub-Grid Scale Stabilization Using Python. *Applied Sciences (Switzerland)*, *15*(18). https://doi.org/10.3390/app151810256

Wang, C. S., Ma, Y. S., Peng, P. A., Zou, C. N., Xie, S. C., Xiao, W. J., Zhang, S. C., Hu, X. M., Wang, J., Gao, S., Hou, M. C., Zhu, X. M., Shao, L. Y., Wu, H. C., Liu, Z. F., Chen, Z. Q., Zhu, R. K., & Chen, X. (2025). A Century Journey from Sedimentology to Sedimentosphere Science: Review and Perspective. *Acta Sedimentologica Sinica*, *43*(5). https://doi.org/10.14027/j.issn.1000-0550.2025.037

Wang, W. (2024). Principles of Machine Learning: The Three Perspectives. In *Principles of Machine Learning: the Three Perspectives*. https://doi.org/10.1007/978-981-97-5333-8

Wei, G., Zheng, B., Dong, J., Yang, Y., Yang, G., Song, S., Guo, S., & Qi, S. (2025). Research Progress of Three-Dimensional Engineering Geological Evaluation Modeling. In *Sustainability (Switzerland)* (Vol. 17, Issue 8). https://doi.org/10.3390/su17083739

Yu, K., Yao, Y., Zhang, W., Lu, L., Gao, Q., Zhang, P., & Sing, S. L. (2025). Defects, monitoring, and AI-enabled control in soft material additive manufacturing: a review. *Virtual and Physical Prototyping*, *20*(1). https://doi.org/10.1080/17452759.2025.2588456

Yu, R., & Wang, R. (2024). Learning dynamical systems from data: An introduction to physics-guided deep learning. *Proceedings of the National Academy of Sciences of the United States of America*, *121*(27). https://doi.org/10.1073/pnas.2311808121

Zamlynskyi, V., Kalinichenko, S., Kniazkova, V., Skrypnyk, N., & Avriata, A. (2025). Healthcare management using artificial intelligence. In *Revolutionizing Medical Systems Using Artificial Intelligence: A Breakthrough in Healthcare*. https://doi.org/10.1016/B978-0-443-32862-6.00001-8

Zarubin, V. I., & Andreeva, I. v. (2025). Integrated process-project model of functioning of a secondary comprehensive educational organization. *Vestnik Majkopskogo Gosudarstvennogo Tehnologiceskogo Universiteta*, *2*. https://doi.org/10.47370/2078-1024-2025-17-2-81-98

Zhao, X. W., Tong, X. M., Ning, F. W., Cai, M. L., Han, F., & Li, H. G. (2025). Review of empowering computer-aided engineering with artificial intelligence. *Advances in Manufacturing*. https://doi.org/10.1007/s40436-025-00545-0

Zhu, C. (2020). Research on mathematical modeling strategy based on MATLAB from the perspective of innovative ability training. *Journal of Physics: Conference Series*, *1648*(2). https://doi.org/10.1088/1742-6596/1648/2/022017

Zhu, T., Wu, C., Zhang, Z., Li, Y., & Wu, T. (2025). Research on Evaluation Methods of Complex Product Design Based on Hybrid Kansei Engineering Modeling. *Symmetry*, *17*(2). https://doi.org/10.3390/sym17020306

Равочкин, Н. Н. (2021). SOCIAL IDEAS DISTRIBUTION IN CONTEMPORARY SOCIETY. *Вестник Тверского Государственного Университета. Серия: Философия*, *1*(55). https://doi.org/10.26456/vtphilos/2021.1.031

Туробов Алексей, Чумакова Мария, & Вечерин Александр. (2019). Международный опыт применения математико статистических алгоритмов прогнозирования преступности. *Международные Процессы*, *17*(4).

This edition of "*General methodological proposals for Computer Science research: Prototyping, mathematical modeling, and the scientific method",* was completed in the city of Colonia del Sacramento in the Eastern Republic of Uruguay on October 2, 2025

EST. 2021 EMC

EDITORIAL MAR CARIBE

# GENERAL METHODOLOGICAL PROPOSALS FOR COMPUTER SCIENCE RESEARCH:

## Prototyping, mathematical modeling, and the scientific method

MARIA MAURA CAMAC TIZA - ERIKA JUANA ZEVALLOS VERA - OMAR TUPAC AMARU CASTILLO PAREDES - JUAN FRANCISCO RAMÍREZ VELIZ - ISMAEL EDWIN SALAZAR VILLAVICENCIO - JOSÉ CÉSAR PIEDRA ISUSQUI - LUIS ALBERTO JUSCAMAYTA RAMÍREZ

ISBN 978-9915-698-39-7